A Sectoral Model Extension to STREAM

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Abstract

This paper proposes a sectoral extension to STREAM, the Bank's main macroeconometric model. The extension documented in this paper borrows heavily from linking integration strategies prevalent in literature. The approach utilised here however, differs in two main ways when compared to existing integrated models. Unlike other integrated models the model proposed in this paper makes use of three modules and two different integration strategies, making it extremely flexible and able to answer a range of policy oriented questions. Secondly, this model utilises a fully-fledged macroeconometric model in its EC module, allowing for more realistic dynamics when compared to the single equation EC models used in literature.

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Contents

1 Introduction 2

2 Motivations for integrated models 3
  2.1 Improved forecast performance 4
  2.2 Enhanced impact analysis 5
  2.3 A more comprehensive model evaluation 5

3 Type of integration strategies 6
  3.1 Embedding strategy 6
  3.2 Linking strategy 7
  3.3 Coupling strategy 8

4 Model description 10
  4.1 Selection of integration strategy 10
  4.2 Disaggregating final demand results - EC → IO link 12
  4.3 Aggregating sectoral results 16

5 Simulations 24
  5.1 Aggregate shocks 24
  5.2 Sectoral shocks 29

6 Conclusion 37
1 Introduction

In the last decades, Central Banks have been responsible for maintaining price stability and in the process, stabilising output fluctuations along the business cycle. Understandably, modelling activities within Central Banks have been heavily influenced by this mandate. Broadly speaking, Pagan (2003) sorts Central Bank models into two main categories, those with a strong focus on structural foundations, containing simultaneous equation models within a general equilibrium framework, and models with a strong emphasis on data matching, mostly comprising of time series models. Both types of models focus on explaining fluctuations in aggregate demand and price levels, with very little regard to sectoral developments.1

Within the past decade, the Central Bank of Malta has overhauled its entire suite of models comprising of traditional econometric models (see Grech et al. (2013), Grech and Micallef (2014), Grech and Rapa (2016), Borg et al. (2019), Micallef and Debono (2020)), Bayesian VARs (Borg and Ruisi, 2018 and fully structural models (Rapa (2016), Rapa (2017) and Gatt et al. (2020). While varying considerably in their degree of theoretical consistency and support to the data, all these models are particularly suited at analysing developments in aggregate levels of output and prices at business cycle frequencies. Also, as is common with policy models in use in other Central Banks, all tools within the suite of models available at the Central Bank of Malta offer little or no information about sectoral developments. From a practical perspective, this has two implications. First, the models currently available within the Bank are unable to provide information on sectoral developments following aggregate shocks. Secondly, macroeconomic models are not well suited to understand the implications of a sector specific shock. This latter point is especially important in understanding the implications a sector specific shock can have both on other sectors, through direct, indirect and induced effects, as well as on aggregate economic activity. Such information is especially useful to analysts and forecasters alike, who wish to internalise the effects of sector specific developments on the aggregate economy.

Against this backdrop, this paper proposes a sectoral extension to STREAM, the Bank’s main macroeconometric model (Grech and Rapa, 2016). The choice of extending STREAM over

1Recently Central Banks have also been utilising Commutable General Equilibrium (CGE) models which as the name implies are of a General Equilibrium type and offer a varying degree of sectoral disaggregation. Still these models are not, at least for the time being, the workhorse models used by Central Banks both in their simulation and forecasting processes.
MEDSEA (Rapa, 2016) is based on the greater flexibility of the former model allowing it to be useful for a larger spectrum of applications. Moreover, STREAM is nowadays an integral part of the Bank’s macroeconomic forecasts, implying that this extension can also be useful from a forecasting perspective, whenever the forecaster wishes to internalise sector-specific information within the forecasts in a more complete and transparent way. The sectoral extension documented in this paper borrows heavily from model integration methods which are prevalent in regional economics literature (see Rey, 1997, Rey, 1998, Rey, 1999 and Fritz et al., 2003), as well as from models used in a national context (see Preston, 1975, Conway, 1990 and Santiago et al., 2011). This extension however, differs in two main ways when compared to other approaches used in literature. First, unlike other models which utilise one type of integration strategy, the approach documented in this paper contains three different modules which allow for different integration links. Secondly, the model proposed here utilises a fully-fledged macroeconometric model in its EC module, which is considerably richer in terms of channels, allowing for more realistic dynamics when compared to the single equation EC models used in literature. Moreover, the approach being proposed in this paper varies considerably from other EC+IO models used in Malta, (see for instance STEMM (EPD, 2019), providing a new way in which macroeconomic relations and disaggregated information can be combined for the Maltese scenario.

The rest of the paper is structured as follows. Section 2 looks at the motivations behind model integration, section 3 provides a short review of the approaches adopted in literature while section 4 describes the model integration approach adopted. Section 5 shows the properties of the model while section 6 concludes.

2 Motivations for integrated models

Broadly speaking, model integration involves the merging of two models, most often an error-correction (EC) model with an input output (IO) model. The theoretical motivations behind the integration of these two types of models is quite obvious when one looks at their characteristics. An EC model is dynamic in nature, and has a more comprehensive treatment of theory. Moreover, EC models often depict the economy in a partial disequilibrium context, with a focus on the dynamic adjustment of the economy. Despite their partial equilibrium nature, EC models are still able to partially capture price adjustments in instances where demand is not equal to supply, thereby allowing markets to partially clear. On the other hand, IO models are
static; they do not provide any information on the trajectory of variables from their baseline to a new steady state. Contrary to EC models, IO models are general equilibrium in nature, where sectoral and aggregate demand must equal the supply of primary and intermediate inputs. Despite this feature, most of these models do not provide any information on price movements. Rather, equilibrium is achieved through one-for-one adjustments in demand following supply disturbances and vice versa, with prices playing no role in these adjustments. IO models make up for this apparent lack of theoretical consistency with the neo-classical thinking, by allowing for a detailed sectoral disaggregation; a feature which is typically absent from most EC models as well as from other types of macroeconomic models including Dynamic General Equilibrium ones.

As argued by L’Esperance (1981), another theoretical advantage of integrated models stems from their ability to allow for a more realistic treatment of final demand. This is especially true when compared to stand-alone IO models, which have a very restrictive treatment of final demand fluctuations. Indeed in most IO models, final demand is often treated as exogenous to developments in the economy. Usually the only exception is household consumption which can be easily endogenised in so-called “closed” IO models. Even in closed IO models however, the link between income and consumption is very basic since this class of models are unable to distinguish between average and marginal propensity to consume, and are unable to capture wealth and liquidity effects while treating both employed and unemployed persons uniformly (Batey and Weeks, 1989). In addition, integrated EC+IO models have been used to relax the fixed employment-output relation that is common in IO models. Therefore, an integrated model is able to combine the diverse, yet complementary characteristics of IO and EC models into a single tool. In other words, integrated EC+IO model provides a more theoretically rich treatment of shocks (similar to an EC model) while at the same time providing a sectoral disaggregation of results (in line with and IO model).

Apart from these theoretical motivations, Rey (1999) argues that there are a number of practical motivations behind combining IO and EC models. Broadly speaking, these motivations fall in three categories: improved forecast performance, enhanced impact analysis capabilities and simpler model evaluation.

2.1 Improved forecast performance

Moghadam and Ballard (1988) and Rey (1998), have argued that integrated EC+IO models
offer more accurate forecasts then traditional semi-structural models. In addition, Rey (1999) suggests that information contained in IO models can be used to set prior restrictions on VAR coefficients, either in a frequentist setup or in Bayesian approach through the setup of prior distributions, which help increase the efficiency of the estimator used to estimate these models, resulting in improved inferences and in increased forecasting precision.

2.2 Enhanced impact analysis

The main motivation behind integrating IO and EC Models is the ability of such models to enhance the scenario analysis capabilities, over what is usually achievable with either IO or EC models on their own. A long known problem of simulations based on IO models, is that any results will be static in nature. This means that these models are unable to capture how long will shocks take to propagate within the sectors of an economy. This is especially problematic for short-term results following shocks. With no information on dynamics, more specifically on the degree of inertia in variable responses to shocks, IO models tend to overestimate the true impacts in the short-to-medium run. Due to their short-run data driven part together with their error correction process around a cointegrating relation, EC models allow for significantly more realistic dynamics. On the other hand, due to their aggregate nature, EC models are unable either to simulate shocks to specific industries or to provide disaggregated responses following aggregate shocks. Another limitation of IO models, is that relative to EC models, they lack any measurement of uncertainty around the estimates provided. In this light, the integration of these two modelling approaches would allow to combine the dynamic capabilities of EC models as well as an ability to provide measurements of uncertainty around point estimates, to a high degree of disaggregation provided by IO models.

2.3 A more comprehensive model evaluation

The third advantage of integrated models over their stand-alone counterparts is that in the former type, model evaluation is more comprehensive. EC models are unable to provide any industry-specific results implying that the model user is unable to judge what the model is implying on the different sectors of the economy. On the other hand, IO model evaluation is usually limited to assessing the reasonableness of output, GVA and employment multipliers. An integrated model, on the other hand, allows the researcher to assess both the implications aggregate results from
Table 1: Taxonomy of Integration Strategies

<table>
<thead>
<tr>
<th>Integration Strategy</th>
<th>Integration Regime</th>
<th>Integration Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Embedding</td>
<td>None</td>
<td>Composite</td>
</tr>
<tr>
<td>Linking</td>
<td>Recursive</td>
<td>Modular</td>
</tr>
<tr>
<td>Coupling</td>
<td>Simultaneous</td>
<td>Composite</td>
</tr>
</tbody>
</table>

Table provides a summary of the combination of integration regimes and structures consistent with each integration strategy. Source: Rey (1997)

the EC module might have on sectoral responses as well as the reasonableness of IO-derived multipliers, by comparing their implications with observable data, most notably for GVA and employment.

3 Type of integration strategies

Rey (1997) suggests a comprehensive taxonomy of integration strategies that can be pursued, providing a detailed review of each model type and related methodological issues. Broadly speaking, the author identifies three integration strategies: embedding, linking and coupling. These three strategies differ across two dimensions; the integration regime and structure employed. The integration regime characterises the nature and level of integration chosen. The former relates to whether the modules or models are integrated in a sequential (thus specifying a clear direction of causation between the IO and EC parts), or in a simultaneous or two-way fashion. The level or extent of integration, relates to the number of interactions included in the models and is a function of the level of detail (sectoral detail in the IO part and theoretical detail in the EC model) in the two modules. The integration structure relates to the mathematical methods chosen for solving the system. For instance, in composite structures, the two modules are solved using iterative algorithms such as Gauss-Siedel. In modular structures, each module is solved independently prior to any form of interaction between the two.

3.1 Embedding strategy

The integration strategy is given by the specific choices made in devising the integration regime and structure (see Table 1). Embedded models usually follow no integration regime, since IO information is embedded within the EC module. Since the there are no modules within such a
model, an embedded model is said to possess a composite structure. Embedding models are quite popular in the development of regional employment models such as in Moghadam and Ballard (1988). Integrated models based on an embedding strategy are dominated by their EC module in the sense that IO tables are used to provide prior information in the form of coefficient restrictions. From an econometric perspective, the estimation of a sectoral error-correction equations (and regional equation in regional models) is often seen as unfeasible. To aid in the estimation process, IO information is used to subsume a variable of interest, most often sectoral employment, with a more aggregated variable derived using IO inter-industry coefficients. As demonstrated in Rey (1998), misspecification of the restrictions in these types of model is quite common resulting in a considerable loss in forecast and simulation performance. The latter is due to the fact that the nature of these restrictions has been commonly ignored in literature leading to a mis-interpretation of inter-industry linkages. In this light, embedding strategies are usually regarded as being a less-comprehensive method of integration when compared to the linkage and coupling strategies (see Rey, 1999 and Fritz et al., 2003).

3.2 Linking strategy

Linking strategies make more extensive use of the information contained in each part of the model. Broadly speaking, in a linking strategy, the output of one module is used as an exogenous input to the second module. While the vast majority of models integrated via linking strategy are modular and recursive in nature (in line with the taxonomy of Table 1), it is possible to envisage a model which is modular in structure yet simultaneous in its interaction regime, such as in Isard and Anselin (1982). Linking strategies have been accomplished in two different ways. In the EC → IO type of linking, the analyst endogenously produces a set of aggregate final demand shocks (most often by specifying an error-correction equation for each final demand component), which are then fed to the IO module (see L’Esperance, 1981). The direction of this recursion is inverted in the other type of linking, that is in the IO → EC linking strategy. After having specified sector specific shocks, results are used to shock a series of sectorally disaggregated error-correction equations. It is important to note that these two linking strategies differ extensively in the level of data requirements required. Indeed, the number of error-correction equations, and consequently data requirements, are larger in the case of IO → EC linking, since this strategy requires the estimation of an error-correction equation for each industry.
### Table 2: Domestic Input Output Table

<table>
<thead>
<tr>
<th>Inter-Industry Demand</th>
<th>Final Demand</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_{1,1}$ $z_{1,2}$ ... $z_{1,S}$</td>
<td>$c_1$ $g_1$ $i_1$ $ii_1$ $x_1$</td>
<td>$y_1$</td>
</tr>
<tr>
<td>$z_{2,1}$ $z_{2,2}$ ... $z_{2,S}$</td>
<td>$c_2$ $g_2$ $i_2$ $ii_2$ $x_2$</td>
<td>$y_2$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$z_{S,1}$ $z_{S,2}$ ... $z_{S,S}$</td>
<td>$c_S$ $g_S$ $i_S$ $ii_S$ $x_S$</td>
<td>$y_S$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Imports</th>
<th>GVA*</th>
<th>Taxes - subs</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1$ $m_2$ ... $m_S$</td>
<td>$m_c$ $m_g$ $m_i$ $m_{ii}$ $m_x$</td>
<td>$ts_1$ $ts_2$ ... $ts_S$</td>
<td>$y_1$ $y_2$ ... $y_S$</td>
</tr>
<tr>
<td>$v_1$ $v_2$ ... $v_S$</td>
<td>$ts_c$ $ts_g$ $ts_i$ $ts_{ii}$ $ts_x$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*At basic prices

### 3.3 Coupling strategy

The most ambitious integration strategy is the coupling strategy, where the researcher allows for a simultaneous relation to exist between the IO and EC modules. In order to better understand this strategy, let us first illustrate the blocks that make up a general input output table. This illustration will also be useful later on when documenting the model proposed in this paper in section 4. The implications of the IO table shown in table 2, can be summarised in a set of linear equations:

\[
y_i = \sum_{j=1}^{S} z_{i,j} + \sum_{k=1}^{K} f_{i,k}^d \quad \forall j = 1, \ldots, S.
\]  

(1)

where $y_i$ is output in industry $i$, $z_{i,j}$ is the output demanded by industry $j$ and supplied by sector $i$ and $f_{i,k}^d$ is the $k^{th}$ domestic final demand component of industry $i$. For instance when $k = 1$ and $i = 1$, $f_{1,1}^d = c_1$. Similarly the $3^{rd}$ type of final demand component for the $4^{th}$ industry will be denoted by $i_4$.

In matrix form, the above relation can be written as:

\[
y = Ay + f^d;
\]  

(2)
where \( y \) is a vector containing sectoral output, \( A \) is a matrix of technical coefficients computed as: 
\[
a_{i,j} = \frac{z_{i,j}}{y_j},
\]
such that \( a_{i,j} \) can be interpreted as the ratio of the inputs produced by sector \( i \) purchased by industry \( j \), \( z_{i,j} \), to the total input used by sector \( j \), \( y_j \). \( f^d \) is a vector of total final demand observed for each industry, \( \Sigma_{k=1}^{K} f_{i,k}^d \).

In order to achieve simultaneity between the EC and IO modules, models integrated via coupling strategies usually allow for a feedback mechanism within the EC module. Moreover, a major advantage of coupled systems (see Conway, 1990 and Fritz et al., 2003), is that they normally allow for a relaxation of the fixed proportions assumption within the intermediate requirement matrix denoted by elements \( z_{i,j} \) in table 2. Coupled models add an additional layer between output predicted by the IO module or "predicted output" and actual output. "Predicted output" is simulated output consistent with the technological matrix implied by the input output table at a given base-year. Basically, predicted output for industry \( i \), \( \tilde{y}_{i,t} \), can be derived from equation 1 and is given by

\[
\tilde{y}_{i,t} = \Sigma_{i=1}^{S} a_{i,j} y_{i,t} + \Sigma_{k=1}^{K} a_{i,k} f_{k,t}^d \tag{3}
\]

where \( a_{i,j} \) are technical coefficients defined as in equation 2 and \( a_{i,k}^d \) is the fraction of final demand component \( k \) to be delivered to industry \( i \) \( (\frac{f_{i,k}^d}{\Sigma_{k=1}^{K} f_{i,k}^d}) \). Actual output \( y_{i,t} \) is then regressed against "predicted output" \( \tilde{y}_{i,t} \), such that:

\[
y_{i,t} = f(\tilde{y}_{i,t}) \tag{4}
\]

with the estimated coefficient, in theory capturing changes in input-output coefficients over time.

The last building block of a coupled model is a behavioural equation for the determination of each final demand component. Assume for illustrative purposes that equation 5 is a behavioural equation for the determination of final household consumption, where \( W_t \) is the error-correction part of the equation containing the usual determinants of aggregate consumption and \( VA_t \) is aggregate value added.

\[
C_t = f(W_t) + \beta_V A_t + \epsilon_t \tag{5}
\]
A shock in equation 5, will change final household consumption $C_t$ (or $f_{1,t}$), which will then change each industry's "predicted output" in equation 3, in turn implying a change in actual output by equation 4. The change in actual output is then translated into changes in value added, $v_{i,t}$, by the following equation:

$$\Delta v_{i,t} = av_{i,k} \Delta y_{i,t}$$ (6)

where $av_{i,t} = \frac{v_i}{y_i}$ is the value added absorbed in output for industry $i$. The change in value added is then entered back into the behavioural equation 5 as an additional shock.

Basically a coupled system is an algorithm that iterates between equations 5, 3, 4, 6 and back to 5, until the system converges.

4 Model description

4.1 Selection of integration strategy

The selection of the integration strategy for our model was based on two principles. First, the integrated model is required to provide a detailed breakdown of results in order to enhance the simulation and forecasting capabilities of the existent suite of models of the Bank, without altering their existent simulation properties. Secondly, from a practical perspective the extension chosen needs to be feasible especially in the light of a lack of reliable timeseries data for sectoral variables, most notably, sectoral GVA deflators. This paper proposes an extension to STREAM, the Bank’s macroeconometric model. STREAM, is the natural candidate for this extension, considering its greater flexibility when compared to the other macro-models in use at the Bank, making it able to be used for a large spectrum of applications, including scenario analysis and forecasting.

With regards to the actual integration strategy, an embedding strategy is quite a loose type of integration strategy, which simply internalises sectoral information from IO tables as restrictions in the estimation procedure of the EC model. Such a strategy would not be particularly useful for increasing the disaggregation level of our existent models. Moreover, an embedding strategy will by its very nature change the estimated coefficients of the EC module. On the other side of
the spectrum, a coupling strategy, is a very extensive integrating strategy which however requires substantial modifications to the EC module, in our case of STREAM. Moreover, it requires time series data of real sectoral GVA (in line with equations 6 and 5). Moreover, it would require a re-estimation of STREAM, which would surely result in a change in its simulation properties.

In this light, a linking strategy is believed to provide the correct balance between the extent of the integration between the two models, and its feasibility. In this paper, we choose an EC → IO linking strategy where the order of recursion flows from the EC model to the IO module. Such an integrated model allows for aggregate shocks or forecasts to be decomposed into sectoral results using information contained within an Input Output table. An IO → EC link is also particularly useful if the analyst or forecaster possesses sectoral information and wishes to understand how this might impact aggregate final demand results. Unfortunately, such a linking strategy requires an error correction equation to be estimated for each industry. This poses significant data requirements, some of which cannot be fulfilled with official sources (for instance due to the absence of sectoral GVA price deflators). To this end, we propose a novel strategy which uses two IO modules a Leontief demand model and a Ghoshian supply model. The two modules allow for the simulation of sector-specific demand or supply-side shocks and to capture either backward or forward linkages. Moreover, by utilising information found solely within input output tables, these two modules are able to produce aggregate results for final demand components, and thus mimic the results of a more standard IO → EC link. These results can then inform the analyst in calibrating the required shocks or add-factors in STREAM, such as to match the final demand results produced by this module.

The econometric module used for this integrated model is the third version of STREAM (Grech and Rapa, 2016) re-estimated in line with Borg et al. (2019). Data for the IO module is derived from the latest set of Input output tables, those for the base year 2010, which were published by NSO in 2015. The IO module uses the maximum sectoral disaggregation that has been made publicly available by the statistical office, and therefore provides results for up to 40 different NACE categories.²

²The model can easily provide a lower level of disaggregation.
4.2 Disaggregating final demand results - EC → IO link

This part of the integration is very similar to those found in literature, such as in Santiago et al. (2011). Still, it differs significantly in the EC module which in our case is a fully-fledged macroeconomic model, as opposed to single behavioural equations usually used in literature. This implies that the level of theoretical detail as well as the number of channels captured by the EC module is significantly more comprehensive than in single equations counterparts. Moreover, having such a detailed analytical framework in the EC module helps in providing internally consistent forecasts or impact analysis results, of the different final demand components making up GDP.

To execute the EC → IO link, we take the percentage point deviations for each final demand component $k$ and each time period $t$ as produced by STREAM following a forecasting or simulation exercise, and store it in a $T \times K$ matrix, $S^p$. Since input output tables are published with a considerable lag, their base year will almost certainly never coincide with the final year of estimation of STREAM. We therefore re-base the shocks to millions of euros using information in millions contained in the input output tables using the following expression:

$$S^m = S^p \hat{F}^d$$

where $S^m$ is a $T \times K$ matrix containing the shocks in millions to each final demand component $k$ and each time period $t$, and $\hat{F}^d$ is a diagonal transformation of $f^d$, which is turn defined as in equation 2.\(^3\)

Each row vector of $S^m$, represents the shocks in millions for all $K$ elements of final demand in a given time period $t$. Up to this point these shocks contain no sectoral information. To produce sectoral shocks we decompose each row of $S^m$ by a weighting matrix $H$. The latter is computed using the proportion of each final demand component $k$, that is demanded in each industry $i$. That is each component of matrix $H$, denoted by $h_{i,k}$ is given by $h_{i,k} = \frac{f_{i,k}}{\sum_{i=1}^{K} f_{i,k}}$.

\(^3\)The diagonal transformation is given by $\hat{F}^d = \sum_{i=k}^{K} e'_k f^d e_k e'_k$, where $e_k$ is the k-th basis of $\mathbb{R}^K$. 

12
where $\Sigma_{i=1}^{S} h_{i,k} = 1, \forall k = 1, \ldots, K.$

\[ S = H * S^{m'} \]  \hspace{1cm} (8)

\[
\begin{pmatrix}
    s_{1,1} & s_{1,2} & \cdots & s_{1,T} \\
    s_{2,1} & s_{2,2} & \cdots & s_{2,T} \\
    \vdots & \vdots & \ddots & \vdots \\
    s_{S,1} & s_{S,2} & \cdots & s_{S,T}
\end{pmatrix}
= 
\begin{pmatrix}
    h_{1,1} & h_{1,2} & \cdots & h_{1,K} \\
    h_{2,1} & h_{2,2} & \cdots & h_{2,K} \\
    \vdots & \vdots & \ddots & \vdots \\
    h_{S,1} & h_{S,2} & \cdots & h_{S,K}
\end{pmatrix}
\begin{pmatrix}
    s_{1,1}^{m} & s_{1,2}^{m} & \cdots & s_{1,K}^{m} \\
    s_{2,1}^{m} & s_{2,2}^{m} & \cdots & s_{2,K}^{m} \\
    \vdots & \vdots & \ddots & \vdots \\
    s_{T,1}^{m} & s_{T,2}^{m} & \cdots & s_{T,K}^{m}
\end{pmatrix}^{t}
\]

$S$ is a matrix in which each column contains the shock in millions that is attributed at a given $t$ to each industry $i$. Therefore we can think of $H$ as a matrix that transforms a set of time varying shocks to final demand, $S^{m}$, into a set of sector-specific shocks in each point in time.

Sectoral final demand changes are then inputted as exogenous shocks within a Leontief demand-driven model which then produces both aggregate and sector specific changes in total demand.

This can be directly derived from equation 2 by solving for total output $y$.

\[
y = Ay + f^{d}
\]

\[
y = (1 - A)^{-1} f^{d}
\]  \hspace{1cm} (9)

where $L = (1 - A)^{-1}$ is the Leontief inverse, where each element $l_{i,j}$ measures the value of production generated directly and indirectly in sector $i$, per each unit of final demand absorbed in sector $j$. Differentiating with respect to $f^{d}$, we get:

\[
\Delta y = L \Delta f^{d}
\]  \hspace{1cm} (10)

Now we let $\Delta f$ be equal to the time-varying final demand shocks that have been apportioned
across all industries, denoted by $S$:

\[
\Delta Y = LS
\]

where $\Delta Y$ is a matrix in which each row $i$ contains the change in output in industry $i$ at different time periods $t$. The aggregate change in output estimated in a given time period $t$, is given by:

\[
\Delta y_{i,agg}^t = \sum_{i=1}^{S} \Delta y_{i,t}
\]

Equations 11 and 12, show the change in output or production that is triggered by a change in final demand. However, most often a policymaker is more concerned about changes in value added (which is more relatable to GDP in the national accounts context), household income or employment. The derivation of these results requires the computation of the respective row vectors of value added input coefficients, labour income coefficients and employment output ratios, which measure the amount of value added, labour income and employment demand generated for each unit of output by each sector of the economy. These are given by:

\[
av_i = \frac{va_i}{y_i}
\]

\[
ah_i = \frac{h_i}{y_i}
\]

\[
ace_i = \frac{e_i}{y_i}
\]
In order to estimate the sectoral change in GVA, labour income and employment triggered by a change in final demand, we need to transform the elements in the Leontief matrix with information contained in \( av_i \) and \( ah_i \) respectively:

\[
l^v_{i,j} = av_i l_{i,j} \tag{16}
\]

\[
l^h_{i,j} = ah_i l_{i,j} \tag{17}
\]

\[
l^e_{i,j} = ae_i l_{i,j} \tag{18}
\]

Using the information contained in equations 16, 17, 18, together with the solution to the Leontief problem in equation 11, we can find the sectoral changes in value added, compensation of employees and employment levels through:

\[
\Delta V = LV S \tag{19}
\]

\[
\Delta H = LH S \tag{20}
\]

\[
\Delta E = LE S \tag{21}
\]

The elements of each row \( t \) of the three matrices derived in equations 19, 20 and 21, show the sectoral changes in value added, compensation of employees and employment that occur at time \( t \). In line with equation 12, we can find the aggregate change in value added, labour income and

\footnote{These results are implicitly based on Simple Leontief multipliers, in the sense that they reflect only direct and indirect effects and omit the effects derived from an increase in labour income (i.e. induced effects). In case the researcher requires to endogenise household consumption by internalising induced effects, this can be done by solving the same system using a closed Leontief model. This entails augmenting matrix \( A \) in equation 9 with labour income and household consumption. Such a system, based on so-called total multipliers, would then be able to capture the inter-relationships between revenue, income, and expenditure flows made by households and the productive sector (Cassar, 2015). However, solving this integrated system using an augmented Leontief demand structure is not recommended. STREAM, the EC module at the heart of this integrated model, contains complex channels that allow to capture income effects which are similar to the induced effects captured by an augmented Leontief model. Thus the changes in final demand produced by STREAM and consequently inputted in the IO module, already include endogenous changes in household consumption. In this light, solving the IO system using a closed Leontief model would most likely result in overestimating household income or induced effects.}
employment at a given time period $t$ by summing across rows as follows:

$$\Delta v_{i,t}^{agg} = \sum_{i=1}^{S} \Delta v_{i,t}$$  \hspace{1cm} (22)$$

$$\Delta h_{i,t}^{agg} = \sum_{i=1}^{S} \Delta h_{i,t}$$  \hspace{1cm} (23)$$

$$\Delta e_{i,t}^{agg} = \sum_{i=1}^{S} \Delta e_{i,t}$$  \hspace{1cm} (24)$$

Due to publication delays associated with input output tables, the base year for the IO module will most probably not be the same as the final year of estimation in STREAM. In order to ensure comparability in the results of the IO and EC modules, results for equations 11 - 12 and 19 - 24 are translated into percentage deviation from baseline levels.\(^5\)

$$\Delta y_{i,t}^P = \frac{\Delta y_{i,t}}{y_i} \times 100; \quad \Delta v_{i,t}^P = \frac{\Delta v_{i,t}}{v_i} \times 100; \quad \Delta h_{i,t}^P = \frac{\Delta h_{i,t}}{h_i} \times 100; \quad \Delta e_{i,t}^P = \frac{\Delta e_{i,t}}{e_i} \times 100;$$

$$\Delta y_{t}^{P,agg} = \frac{\Delta y_{t}^{agg}}{\sum_{i=1}^{S} y_{i}} \times 100; \quad \Delta v_{t}^{P,agg} = \frac{\Delta v_{t}^{agg}}{\sum_{i=1}^{S} v_{i}} \times 100; \quad \Delta h_{t}^{P,agg} = \frac{\Delta h_{t}^{agg}}{\sum_{i=1}^{S} h_{i}} \times 100; \quad \Delta e_{t}^{P,agg} = \frac{\Delta e_{t}^{agg}}{\sum_{i=1}^{S} e_{i}} \times 100;$$

\(4.3\) Aggregating sectoral results

As discussed in section 3.2, the most common way to produce an IO $\to$ EC link, is to first estimate the results for sectoral shocks within the IO module, and then input them into sectoral error-correction equations. In light of the type of EC module used in this integrated model (which is of a macro-econometric type), such a strategy is not feasible. Instead, in order to

\(^5\)The different nature of the IO and EC modules together with differences in the IO base year and estimation year of STREAM, imply that the percentage deviation of aggregate value added and the percentage deviation in GDP consistent with the shocks inputted in matrix $S^P$ need not be equal. This might result in slight consistency issues between the aggregate final demand results provided by STREAM and the disaggregated results provided by the IO module. To this end, the user can force the aggregate percentage change in GVA implied by the IO module to equate the percentage deviation in GDP implied by STREAM. This is done by a simple and linear rescaling all shocks contained in $S^P$. 

16
aggregate responses derived from disaggregated shocks, we choose to rely solely on information contained within the Input Output tables. This means that technically speaking, this part of the model is not an IO → EC link. However, it can still turn out to be particularly useful whenever the researcher possesses information on shocks that are expected to hit particular industries. Moreover, unlike other contributions in literature, the integrated model presented in this paper, makes use of two different IO modules, one based on the Leontief demand driven model (in line with that used in the EC → IO link), and one based on the Ghoshian model of supply. This allows this part of the integrated model to capture both upstream (demand-side) and downstream (supply-side) shock propagation depending on the type of shock required.

4.3.1 Sectoral results following demand and supply-side shocks

The core equations used to produce demand-side shocks disaggregated by sector are similar to those used in the EC → IO link in section 4.2. This part of the model basically requires the user to input information directly in $S$, thus bypassing equation 8. Sectoral results for output, value added, labour income and employment are then estimated using equations 11, 19, 20 and 21.

A different module is however required if the user is interested in estimating the effects sectoral supply shocks might have on sector-specific variables of interest, such as GVA. A demand shock relies on backward linkages to trace the shock propagation. That is, following a final demand shock, the Leontief demand driven model traces which industries are responsible in supplying intermediate production to the sector which is being shocked. The propagation mechanism of a supply shock, on the other hand, works through the forward linkages of a particular sector. Forward linkages capture the links any given sector has with downstream sectors. A change in the primary inputs of sector $j$ implies a change in the amount of product $j$ that is available to be used as intermediate inputs by all other sectors. Thus in a Ghoshian model, total forward linkages of sector $j$ are measured as the change in the output of all other sectors that occurs due to a change in the inputs used by sector $j$ Miller and Blair (2009). In order to better capture these effects this module relies on the Ghoshian supply model which is characterised by the following relations:

$$y_j = \sum_{i=1}^{S} z_{i,j} + \sum_{l=1}^{L} \nu_{j,l} \tag{25}$$
where \( v_{j,k} \) is the \( l^{th} \) type of primary input, including Imports, Gross Value Added (which is the sum of Gross Operating surplus and labour income) and taxes less subsidies. Going back to table 2, when \( l=3 \) and \( i=1 \), \( v_{i,k} \) will refer to \( ts_1 \), or taxes and subsidies of sector 1.

In matrix form, the above relation can be written as:

\[
y' = y' B + v'
\]  

(26)

where \( y' \) is the transpose of the same vector containing sectoral output in equation 2, \( v \) is a vector of primary inputs, \( B \) is a matrix of allocation coefficients (as opposed to technical coefficients as in the case of the Leontief model) computed as: \( b_{i,j} = z_{i,j}/y_j \), such that \( b_{i,j} \) can be interpreted as the distribution of sector \( i \)'s, output across sectors \( j \) that purchase interindustry outputs from \( i \).

The model can be easily solved in a similar fashion to the Leontief model:

\[
y' = v'(1 - B)^{-1}
\]

(27)

where \( G = (1 - B)^{-1} \) is the Output inverse where \( g_{i,j} \) measures the total value of production that comes about in sector \( j \) per unit of primary input in sector \( i \).

Differentiating with respect to \( v' \):

\[
\Delta y' = (\Delta v')G
\]

(28)

In line with equation 11, we can express the above as a function of a set of time varying shocks to sectoral primary inputs, contained in a \( T \times S \) matrix, \( S^G \),

\[
\Delta Y^G = S^G G
\]

(29)

where \( \Delta Y^G \) is a \( T \times S \) matrix in which each element \( \Delta y^G_{i,j} \) contains the change in output in industry \( j \) at time \( t \). We then find the sectoral change in GVA, labour income and employment.
triggered by a change in primary inputs using the following relations:

\[ \Delta V^G = S^G G^V \]  
(30)

\[ \Delta H^G = S^G G^H \]  
(31)

\[ \Delta E^G = S^G G^E \]  
(32)

where elements of \( G^V \) are estimated as \( g_{i,j}^V = g_{i,j} va_j \), those of \( G^H \) as \( g_{i,j}^H = g_{i,j} ah_j \) and \( G^E \) as \( g_{i,j}^E = g_{i,j} ae_j \). We then estimate the aggregate changes in the three variables of interest as:

\[ \Delta v_{t}^{agg,G} = \sum_{j=1}^{S} \Delta v_{t,j}^G \]  
(33)

\[ \Delta h_{t}^{agg,G} = \sum_{j=1}^{S} \Delta h_{t,j}^G \]  
(34)

\[ \Delta e_{t}^{agg,G} = \sum_{j=1}^{S} \Delta e_{t,j}^G \]  
(35)

Finally we estimate these changes as a percent of baseline levels using the following relations:

\[ \Delta y_{t,j}^{P,G} = \frac{\Delta y_{t,j}^G}{y_{i}} \times 100; \quad \Delta v_{t,j}^{P,G} = \frac{\Delta v_{t,j}^G}{v_{i}} \times 100; \quad \Delta h_{t,j}^{P,G} = \frac{\Delta h_{t,j}^G}{h_{i}} \times 100; \quad \Delta e_{t,j}^{P,G} = \frac{\Delta e_{t,j}^G}{e_{i}} \times 100; \]

4.3.2 Estimating changes in final demand aggregates

We then use these results and information contained within the final demand matrix in table 2, to compute changes in aggregate final demand components \( k \) for different time periods \( t \). The estimation of changes in final demand depends on the type of shock. More precisely, under a
demand shock scenario, $\Delta f_{k,t}^d$ is estimated as:

$$
\Delta f_{k,t}^d = \Sigma_{i=1}^S \Delta f_{i,k,t}^d
$$

$$
\Delta f_{i,k,t}^d = \Delta v_{i,t}^P \cdot f_{i,k}^d
$$

where $\Delta f_{i,k,t}^d$ are elements of a 3-dimensional matrix $\Delta F^d$, of size $S \times K \times T$, containing the change in final demand component $k$ at basic prices, in industry $i$ at time $t$ following a demand shock and $\Delta f_{k,t}^d$ is the aggregate change in final demand component $k$ at basic prices at time $t$ following a demand shock.

$\Delta f_{k,t}^{d,G}$, that is the change in final demand component $k$ at time $t$ after a supply-side shock is found by slightly modifying equation 36:

$$
\Delta f_{k,t}^{d,G} = \Sigma_{j=1}^S \Delta f_{j,k,t}^{d,G}
$$

$$
\Delta f_{j,k,t}^{d,G} = \Delta v_{j,t}^{P,G} \cdot f_{k,j}^d
$$

where $\Delta f_{j,k,t}^{d,G}$ are elements of a matrix $\Delta F^{d,G}$ containing changes in final demand component $k$ in industry $j$ and time $t$ after a supply shock, and where $f_{k,j}^d$ contain the final demand component $k$ absorbed by industry $j$, and is thus simply the transpose of the vector containing elements $f_{i,k}^d$ where $i = j$.

These relations implicitly assume that for each industry, the share of different final demand components in total final demand is constant. This implies that the change in final demand type $k$ at basic prices at a specific point in time $t$, given by $\Delta f_{k,t}^d$ and $\Delta f_{k,t}^{d,G}$, depends on the percentage change in the value added of each industry at time $t$, and the relative weight that each final demand component $k$ has in the total final demand of each industry. Thus for instance, the change in aggregate consumption, will be larger, the larger is the change in the value added of industries in which consumption makes up the larger share in total sectoral final demand.

Within an input output framework, the Gross Domestic Product (GDP) can be found by summing up the total use of final demand at purchasers’ prices less total economy imports Claus
Thus, in order to get to a final figure of aggregate GDP change, we need to first find the change in imports that is implied by the change in each final demand component at basic prices, as well as translate the latter into purchasers’ prices.

### 4.3.3 Estimating imports for given final demand changes

In order to construct the import contents of the final demand components, we utilise both information contained in the domestic input output table (Table 2) as well as information contained in the so-called Import Input Output Table. The latter table (an example of which is shown in Table 3), records both inter-industry imports, that is the imports demanded from sector \(i\), needed by sector \(j\) (recorded in matrix \(Z^m\) with elements \(z^m_{i,j}\)), as well as direct imports, that is the imports that are imported by each final demand component (recorded in matrix \(F^m\) with elements \(f^m_{i,k}\)). The computation of macro import intensities requires us to find both direct (\(M^{dir}\)) and indirect imports (\(M^{ind}\)). The former is directly given by \(F^m\):

\[
M^{dir} = F^m
\]  

(38)

A matrix of indirect imports \(M^{ind}\) is a \(S \times K\) matrix, that records for each expenditure component \(k\), the value of indirect imports "induced" in each sector \(i\) by the expenditure of domestically provided goods and services, including imports of intermediate inputs acquired by domestic producers (Bussière et al., 2013). To get to this matrix, we need to internalise both the structures of imports demanded by each industry for intermediate production, contained in matrix \(Z^m\), as well as information on the domestic production process which is contained within the Leontief matrix \(L\) and derived in equation 9. Following Bussière et al. (2013) we first find \(X\), which is a \(S \times K\) matrix of domestic output induced by each expenditure component \(k\) using:

\[
X = LF^d
\]  

(39)

where \(F^d\) is a matrix containing final demand components from the domestic Input Output Table. Secondly, the imports of intermediate output from sector \(i\) induced by the expenditure

21
Table 3: Imports Input Output Table

<table>
<thead>
<tr>
<th>Inter-Industry Imports</th>
<th>Direct Imports</th>
<th>Imports</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_{1,1}^m$ $z_{1,2}^m$ ... $z_{1,S}^m$</td>
<td>$c_1^m$ $g_1^m$ $i_{11}^m$ $x_{11}^m$</td>
<td>$m_1$</td>
</tr>
<tr>
<td>$z_{2,1}^m$ $z_{2,2}^m$ ... $z_{2,S}^m$</td>
<td>$c_2^m$ $g_2^m$ $i_{22}^m$ $x_{22}^m$</td>
<td>$m_2$</td>
</tr>
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and each time period \( t \).

\[
\Delta F = \Delta \tilde{F}^d + \Delta F^m
\]

(43)

where \( \Delta \tilde{F}^d \) and \( \Delta F^m \) contain the change in the domestic and imported final demand expenditure at basic prices for each expenditure type \( k \) and each time period \( t \), respectively. \( \Delta \tilde{F}^d \) is equal to \( \Delta F^d \) (thus given by equation 36) or \( \Delta F^{d,G} \) (thus given by equation 37) depending on whether the shock is a demand or supply-side one. \( \Delta F^m \) is estimated as:

\[
\Delta f^m_{k,t} = \sum_{i=1}^S \frac{f^m_{i,k}}{f^d_{i,k}} \Delta \tilde{f}^d_{k,t}
\]

(44)

where \( \tilde{f}^d_{k,t} \) are elements in \( \tilde{F}^d \). This assumes that the direct imports required by a unit of each domestic final demand component is constant across the time \( t \).

Having found these matrices, it is possible to estimate the change in imports at each time period \( t \) that is induced by a change in each one the \( K \) total final demand components at basic prices:

\[
\Delta M = w \Delta F
\]

(45)

4.3.4 Estimating the change in GDP

The final step before aggregating for GDP, is to estimate domestic final demand in purchaser prices, that is by adding the changes in taxes and subsidies to the change in domestic final demand. Let \( f^d \) and \( ts \) be \( 1 \times K \) vectors containing final demand at basic prices (given by \( \sum_{i=1}^S f^d_{i,k} \)) and taxes and subsidies for \( k \) final demand components in line with table 2, then domestic final demand at purchaser prices, \( f^{d,pp} \), is equal to \( f^d + ts \). Assuming that the proportion of taxes and subsidies for each final demand component is constant across time,\(^7\) we can derive the change in taxes and subsidies and the change in domestic final demand at purchaser prices following a

---

6 This is required since the import intensities are estimated on total final demand components and not on domestic final demand.

7 This assumption is justified by the fact that in the data, the share of taxes less subsidies in each sector’s GVA is relatively stable of time.
shock and across time ($\Delta TS$ and $\Delta F_{d,pp}$ respectively) using:

$$\Delta TS = \frac{l_{sk}}{f_{k}} \cdot \Delta F_{d}$$

$$\Delta F_{d,pp} = \Delta F_{d} + \Delta TS$$

which implies that the percentage change in domestic final demand at basic prices is equal to that in domestic final demand at purchaser prices.

Finally the change in GDP will be given by:

$$\Delta GDP = \Delta F_{d,pp} - \Delta M$$

5 Simulations

This section documents the properties of the integrated model by showing two types of shocks, aggregate shocks which utilise the EC → IO link to provide us with disaggregated results, and disaggregated shocks that use information contained in the Leontief and Ghoshian models to provide us with both disaggregated effects on value added, employment and labour income as well as with a view on aggregate final demand developments. The former type of shocks are very useful when the researcher has information on shocks that occur at a relatively aggregate level, such as final demand shocks or interest rate shocks. The latter modules are useful if the researcher has information on shocks that will only hit particular sectors and wishes to estimate the effects these will have on other sectors through indirect or induced effects, while at the same time assess aggregate developments in final demand components and ultimately in GDP.

5.1 Aggregate shocks

This section illustrates the sectoral developments following two aggregate shocks, one to foreign demand and one to local household consumption preferences. Both shocks are first run through the EC module (STREAM), producing results for aggregate final demand developments. Deviations in aggregate demand components are then loaded in the IO module, which then produces
a disaggregation of results.

5.1.1 Foreign Demand Shock

Table 4: Aggregate Results for a Foreign Demand shock of 1%

<table>
<thead>
<tr>
<th></th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Domestic Product</td>
<td>0.58</td>
<td>0.68</td>
<td>0.59</td>
</tr>
<tr>
<td>Household consumption</td>
<td>0.09</td>
<td>0.40</td>
<td>0.39</td>
</tr>
<tr>
<td>Government consumption</td>
<td>0.21</td>
<td>0.23</td>
<td>0.08</td>
</tr>
<tr>
<td>Gross Fixed Capital Formation</td>
<td>0.27</td>
<td>0.64</td>
<td>0.57</td>
</tr>
<tr>
<td>Exports</td>
<td>1.08</td>
<td>0.87</td>
<td>0.64</td>
</tr>
<tr>
<td>Imports</td>
<td>0.73</td>
<td>0.67</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Results defined as % deviation from baseline levels following a 1% increase in export market volume.

The aggregate demand shock is defined as a permanent 1% increase in foreign demand for Maltese goods and services, in line with that featured in Borg et al. (2019). As shown in table 4, the increase in foreign demand has a positive impact on exports, which in turn increases GDP. The increase in aggregate demand, boosts demand for both factors of production. This raises investment and employment, with the latter causing a rise in average salaries, which in turn boosts household disposable income and eventually private consumption. The increase in aggregate demand also results in some upward pressure in local prices, in turn hurting Malta’s international price competitiveness, resulting in a slowdown in exports in the second and third years of the time period under consideration. In the second and third year of the simulation, increases in household disposable income, and in the demand for investment goods as well as a slowly decaying international price competitiveness, lead to a reduction in the contribution of external demand relative to that of domestic demand in driving overall GDP figures.

When analysing sectoral results, it is important to keep in mind that results derived from an IO module do not simply capture direct effects but also indirect ones. In other words, these results do not merely reflect the exposure of sectors to a particular final demand component that is significantly affected by the shock under consideration, but also the degree of interconnectedness of each sector. Thus, the sensitivity of a sector’s gross value added to a particular shock depends on three factors: first, which final demand components are affected the most by the shock, the weight the particular sector plays in the composition of each final demand component and finally, the sector’s interconnectedness in the local intermediate production process measured in terms
### Table 5: Sectoral Gross Value Added Results for a Foreign Demand shock of 1%

<table>
<thead>
<tr>
<th>Sector</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>01-03 Agriculture, forestry and fishing</td>
<td>0.32</td>
<td>0.56</td>
<td>0.54</td>
</tr>
<tr>
<td>B05-09 F41-43 Mining, quarrying and construction</td>
<td>0.37</td>
<td>0.67</td>
<td>0.64</td>
</tr>
<tr>
<td>C10-33 Manufacturing</td>
<td>0.91</td>
<td>0.88</td>
<td>0.75</td>
</tr>
<tr>
<td>D35E36-39 Electricity, gas, steam ...</td>
<td>0.38</td>
<td>0.59</td>
<td>0.54</td>
</tr>
<tr>
<td>G45-47 Wholesale and retail trade...</td>
<td>0.37</td>
<td>0.61</td>
<td>0.58</td>
</tr>
<tr>
<td>H49-53 Transportation and storage</td>
<td>0.84</td>
<td>0.85</td>
<td>0.73</td>
</tr>
<tr>
<td>I55-56 Accommodation, food services activities...</td>
<td>0.15</td>
<td>0.50</td>
<td>0.52</td>
</tr>
<tr>
<td>J58-63 Information and communication</td>
<td>0.55</td>
<td>0.73</td>
<td>0.66</td>
</tr>
<tr>
<td>K64-66 Financial and insurance activities</td>
<td>1.00</td>
<td>0.93</td>
<td>0.78</td>
</tr>
<tr>
<td>L68 Real estate activities</td>
<td>0.18</td>
<td>0.52</td>
<td>0.54</td>
</tr>
<tr>
<td>M69-75 Professional, scientific and technical...</td>
<td>0.86</td>
<td>0.87</td>
<td>0.75</td>
</tr>
<tr>
<td>N77-82 Administrative and support services</td>
<td>0.71</td>
<td>0.74</td>
<td>0.62</td>
</tr>
<tr>
<td>O84 Public administration and defence</td>
<td>0.27</td>
<td>0.32</td>
<td>0.17</td>
</tr>
<tr>
<td>P85 Education</td>
<td>0.23</td>
<td>0.38</td>
<td>0.29</td>
</tr>
<tr>
<td>Q86-88 Human health and social work activities</td>
<td>0.21</td>
<td>0.32</td>
<td>0.20</td>
</tr>
<tr>
<td>R90-93 Arts, entertainment and recreation</td>
<td>1.09</td>
<td>0.97</td>
<td>0.81</td>
</tr>
<tr>
<td>S94-96 Other service activities</td>
<td>0.17</td>
<td>0.51</td>
<td>0.53</td>
</tr>
<tr>
<td>T97-98U99 Households as employers...</td>
<td>0.10</td>
<td>0.47</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Results defined as % deviation from baseline levels following a 1% increase in export market volume.

Looking at the results in table 5, one can however note that at least in the first year following the start of the foreign demand shock, results are mainly being driven by industries which are largely export oriented. These include Arts, entertainment and recreation (which includes the gaming and betting industry), Financial and insurance activities and Manufacturing. The strength of the results pertaining to the Arts, entertainment and recreation as well as the Financial and insurance activities is solely driven by their contribution in Maltese exports. Indeed, the strength of their backward linkages with the rest of the economy (which can be measured in terms of their value added Simple Leontief multipliers) is quite small, especially with regards to the latter sector. The results for the Financial and insurance sector are especially driven by the inclusion of Special Purpose Entities (SPEs) within ESA 2010 data. Since SPEs are mainly export oriented, the...
sectoral decomposition of export final demand is significantly affected by their inclusion. On the other hand, since SPEs contain very high import content their inclusion reduces the relative magnitude of the local intermediate input requirements for this sector, implicitly weakening the strength of this sector’s interconnectedness (Cassar and Rapa, 2018).

Results for the manufacturing sector hide considerable heterogeneity within its sub-sectors (see table 6). Some sectors, such as the Manufacture of computer and electronic products and Manufacture of textiles and wearing apparel are considerably affected by a foreign demand shocks, ranking second and sixth respectively out of a total of 40 industries. In light of their low inter-industry linkages, results for both these sub-sectors are mainly driven by their significant exposure to the export market. As argued previously, even sectors with a low direct exposure to the export market can be affected by a foreign demand shock. This is mostly seen in the Manufacture of basic metals sector, which has quite a low direct exposure to the export market, but has significant inter-industry ties reflected in the fourth highest GVA multiplier.

**Table 6: Sectoral Gross Value Added Results for Manufacturing sectors in Year 1 following a Foreign Demand shock of 1%**

<table>
<thead>
<tr>
<th>Sector Description</th>
<th>Export Weight</th>
<th>GVA multiplier</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>C10T12 Manufacture of food products, beverages and tobacco</td>
<td>0.81 11</td>
<td>0.49 33</td>
<td>0.36 27</td>
</tr>
<tr>
<td>C13T15 Manufacture of textiles, wearing apparel and leather</td>
<td>0.57 13</td>
<td>0.56 26</td>
<td>1.01 6</td>
</tr>
<tr>
<td>C16 Manufacture of wood and of products of wood except furniture...</td>
<td>0.03 30</td>
<td>0.51 30</td>
<td>0.58 16</td>
</tr>
<tr>
<td>C17-22 Manufacture of paper, chemical products, basic pharmaceutical...</td>
<td>4.03 5</td>
<td>0.50 31</td>
<td>0.97 8</td>
</tr>
<tr>
<td>C23 Manufacture of other non-metallic mineral products</td>
<td>0.08 25</td>
<td>0.53 28</td>
<td>0.42 23</td>
</tr>
<tr>
<td>C24 Manufacture of basic metals</td>
<td>0.02 31</td>
<td>0.87 4</td>
<td>0.92 9</td>
</tr>
<tr>
<td>C25 Manufacture of fabricated metal products, except machinery...</td>
<td>0.18 22</td>
<td>0.50 32</td>
<td>0.65 14</td>
</tr>
<tr>
<td>C26-32 Manufacture of computer, electronic and optical products ...</td>
<td>13.31 3</td>
<td>0.32 39</td>
<td>1.11 2</td>
</tr>
<tr>
<td>C33 Repair and installation of machinery and equipment</td>
<td>0.69 12</td>
<td>0.46 34</td>
<td>0.87 10</td>
</tr>
</tbody>
</table>

Results defined as % deviation from baseline levels following a 1% increase in export market volume. Export weight is estimated as the percentage of export final demand absorbed by each sector. Ranking of results is relative to a 40 sector disaggregation.

Going back to the results in table 4, we can note that in Years 2 and 3 of the simulation, as the
contribution of domestic demand in overall GDP starts to increase, deviations in the GVA of domestically oriented industries starts to rise significantly. This is easily seen in the results for the Wholesale and retail trade sector, in which the increase in GVA climbs from 0.37 to 0.60 by the second year of the simulation. These results also show that the Accommodation and food services sector is not only driven by changes in the external demand, but is also quite sensitive to changes in domestic demand.

5.1.2 Household consumption preference shock

Table 7: Aggregate Results for a temporary household consumption shock of 1% in Q1

<table>
<thead>
<tr>
<th></th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Domestic Product</td>
<td>0.19</td>
<td>0.01</td>
<td>-0.08</td>
</tr>
<tr>
<td>Household consumption</td>
<td>0.77</td>
<td>0.38</td>
<td>0.06</td>
</tr>
<tr>
<td>Government consumption</td>
<td>0.08</td>
<td>-0.01</td>
<td>-0.05</td>
</tr>
<tr>
<td>Gross Fixed Capital Formation</td>
<td>0.13</td>
<td>0.08</td>
<td>-0.06</td>
</tr>
<tr>
<td>Exports</td>
<td>-0.00</td>
<td>-0.02</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

Results defined as % deviation from baseline levels following a 1% increase in private consumption in Q1

An exogenous increase in private consumption increases aggregate demand and therefore GDP. In light of the significant import content of household consumption, the increase in GDP is partially offset by an increase in imports. The increase in output leads to an increase in the demand for labour and capital. The latter is reflected in an increase in Gross Fixed Capital Formation, while the former leads to a temporary tightening in the labour market. This causes an increase in compensation of employees, positively affecting disposable income and propping up private consumption even after the exogenous shock is switched off in Q2. The increase in aggregate demand therefore leads to some pressure on the costs faced by firms, leading to a marginal erosion in Malta’s international competitiveness, leading to a slowdown in exports in the second and third year of the simulation horizon.

Turning to the results in table 8, we can note that the sectoral decomposition of aggregate results yields significantly different conclusions from the ones derived after a foreign demand shock. As expected a-priori, export-oriented sectors such as Manufacturing, Financial and insurance activities and Arts, entertainment and recreation are not significantly affected by local demand
shocks. On the other hand, strong results are registered in the Wholesale and retail sector, Accommodation and food services activities, Real Estate Activities and Other service activities. Driven by strong backward linkages, the Electricity generation sector also registers significant increases in its GVA.

Similar to the previous shock, the level of aggregation shown in table 8, hides a considerable amount of heterogeneity at the sub-sector level. For instance, despite the fact that the overall Manufacturing sector is not affected considerably by a local consumption shock, one can note that the GVA of a particular sub-sector, Manufacturing of food and beverages and tobacco products, is expected to grow by around 0.4% in the first year of the simulation. This result is mainly driven both by the weight of this sector in the sectoral decomposition of household consumption, as well as by its strong backward linkages. Indeed this sector is especially important as a supplier to the Wholesale and retail trade, except of motor vehicles sector (which is directly affected by a consumption shock) and to the Agriculture, forestry and fishing sector (which is an important supplier to the Accommodation and food services activities sector).

5.2 Sectoral shocks

This section illustrates the properties of the two IO modules included in this integrated model, that is the Leontief and Ghoshian models. This part of the model is able to account for sector specific shocks and can provide both disaggregated results in terms of GVA, employment and labour income, as well as estimate shock consistent changes in aggregate final demand components. As discussed in 4.3, due to the type of EC module used, the sectoral results of the IO module are not inputted in the EC module, but instead rely solely on the information contained in the Input Output tables. This strategy implies that unlike in the EC → IO link, we are unable to retrieve information on the propagation of the shocks, implying that results will be static in nature. On the other hand, contrary to the previous section, this link is able to provide information on both demand-side shocks, focusing on downstream or backward linkages, and supply shocks, which take in consideration upstream or forward linkages.

In order to illustrate the properties of this link, this section documents the results of a negative

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8 The sector which is mostly affected by an exogenous consumption shock is Activities of households as employers. However, the significantly small size of the sector implies that its actual contribution to overall GVA growth is very close to zero.

29
Table 8: Sectoral Gross Value Added Results for a temporary household consumption shock of 1% in Q1

<table>
<thead>
<tr>
<th>Sector Description</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>01-03 Agriculture, forestry and fishing</td>
<td>0.38</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>05-09 Mining, quarrying and construction</td>
<td>0.12</td>
<td>0.01</td>
<td>-0.04</td>
</tr>
<tr>
<td>C10-33 Manufacturing</td>
<td>0.09</td>
<td>0.00</td>
<td>-0.05</td>
</tr>
<tr>
<td>D35E36-39 Electricity, gas, steam ...</td>
<td>0.32</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>G45-47 Wholesale and retail trade...</td>
<td>0.33</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>H49-53 Transportation and storage</td>
<td>0.14</td>
<td>0.01</td>
<td>-0.02</td>
</tr>
<tr>
<td>I55-56 Accommodation, food services activities...</td>
<td>0.49</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>J58-63 Information and communication</td>
<td>0.21</td>
<td>0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>K64-66 Financial and insurance activities</td>
<td>0.08</td>
<td>0.00</td>
<td>-0.03</td>
</tr>
<tr>
<td>L88 Real estate activities</td>
<td>0.46</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>M69-75 Professional, scientific and technical...</td>
<td>0.09</td>
<td>0.00</td>
<td>-0.08</td>
</tr>
<tr>
<td>N77-82 Administrative and support services</td>
<td>0.14</td>
<td>0.01</td>
<td>-0.10</td>
</tr>
<tr>
<td>O84 Public administration and defence</td>
<td>0.07</td>
<td>0.00</td>
<td>-0.05</td>
</tr>
<tr>
<td>P85 Education</td>
<td>0.20</td>
<td>0.01</td>
<td>-0.02</td>
</tr>
<tr>
<td>Q86-88 Human health and social work activities</td>
<td>0.16</td>
<td>0.01</td>
<td>-0.03</td>
</tr>
<tr>
<td>R90-93 Arts, entertainment and recreation</td>
<td>0.04</td>
<td>0.00</td>
<td>-0.04</td>
</tr>
<tr>
<td>S94-96 Other service activities</td>
<td>0.47</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>T97-98U99 Households as employers...</td>
<td>0.52</td>
<td>0.03</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Results defined as % deviation from baseline levels following a 1% increase in consumption in Q1.

temporary shock of 10% to the final demand of the manufacturing sector, and a 10% shock to the primary inputs of the same sector.

5.2.1 Shock to manufacturing demand

This shock simulates the interindustry developments following a negative 10% shock to the final demand of the manufacturing sector. Since the IO module contains a disaggregation of the manufacturing sector into nine different sub-sectors, the shock is calibrated such that the share of the final demand of each sub-sector in total manufacturing final demand remains constant. In the case of a demand-side shock, the model allows for two sets of results, one based on Simple or Type 1 multipliers and one based on Total or Type 2 multipliers. The former set of results encompasses the direct and indirect effects of a shock, thus capturing the different rounds of
intermediate production required to satisfy a unit of final demand, but exclude induced effects which are captured in the Type 2 or Total multipliers. Induced effects capture the fact that following a positive shock, increases in intermediate production in each production round, require labour input which in turn raises household income increasing household consumption.\(^9\)

**Table 9: Sectoral Gross Value Added Results for a 10% drop in the final demand of the Manufacturing sector**

<table>
<thead>
<tr>
<th>Sector Code</th>
<th>Sector Description</th>
<th>Simple Direct+Indirect</th>
<th>Total Simple+Induced</th>
</tr>
</thead>
<tbody>
<tr>
<td>01-03</td>
<td>Agriculture, forestry and fishing</td>
<td>-1.32</td>
<td>-2.63</td>
</tr>
<tr>
<td>B05-09 F41-43</td>
<td>Mining, quarrying and construction</td>
<td>-0.27</td>
<td>-0.49</td>
</tr>
<tr>
<td>C10-33</td>
<td>Manufacturing</td>
<td>-8.43</td>
<td>-8.73</td>
</tr>
<tr>
<td>D35E36-39</td>
<td>Electricity, gas, steam ...</td>
<td>-1.41</td>
<td>-2.46</td>
</tr>
<tr>
<td>G45-47</td>
<td>Wholesale and retail trade...</td>
<td>-0.85</td>
<td>-1.94</td>
</tr>
<tr>
<td>H49-53</td>
<td>Transportation and storage</td>
<td>-0.70</td>
<td>-1.17</td>
</tr>
<tr>
<td>I55-56</td>
<td>Accommodation, food services activities...</td>
<td>-0.06</td>
<td>-1.73</td>
</tr>
<tr>
<td>J58-63</td>
<td>Information and communication</td>
<td>-0.23</td>
<td>-0.87</td>
</tr>
<tr>
<td>K64-66</td>
<td>Financial and insurance activities</td>
<td>-0.16</td>
<td>-0.42</td>
</tr>
<tr>
<td>L68</td>
<td>Real estate activities</td>
<td>-0.15</td>
<td>-1.72</td>
</tr>
<tr>
<td>M69-75</td>
<td>Professional, scientific and technical...</td>
<td>-0.30</td>
<td>-0.57</td>
</tr>
<tr>
<td>N77-82</td>
<td>Administrative and support services</td>
<td>-0.34</td>
<td>-0.77</td>
</tr>
<tr>
<td>O84</td>
<td>Public administration and defence</td>
<td>-0.03</td>
<td>-0.10</td>
</tr>
<tr>
<td>P85</td>
<td>Education</td>
<td>-0.15</td>
<td>-0.51</td>
</tr>
<tr>
<td>Q86-88</td>
<td>Human health and social work activities</td>
<td>0.00</td>
<td>-0.35</td>
</tr>
<tr>
<td>R90-93</td>
<td>Arts, entertainment and recreation</td>
<td>0.00</td>
<td>-0.14</td>
</tr>
<tr>
<td>S94-96</td>
<td>Other service activities</td>
<td>-0.14</td>
<td>-1.27</td>
</tr>
<tr>
<td>T97-98U99</td>
<td>Households as employers...</td>
<td>0.00</td>
<td>-1.80</td>
</tr>
</tbody>
</table>

Results defined as % deviation from baseline levels following a 10% drop in manufacturing final demand.

Sectoral GVA results are shown in table 9. As expected, the sector which is projected to be hit hardest by this shock, is the Manufacturing sector, whose GVA is estimated to fall between 8.4% and 8.7%. Excluding induced effects, the next highest effect is expected to be recorded in the

\(^9\)Contrary to the EC → IO link, the link described in this section does not make use of STREAM, implying that income effects (which are very similar in principle to induced effects) are not being captured implicitly in these modules. Thus, the use of Total multipliers in this case is theoretically possible, should the user be interested in capturing induced effects.
Electricity generation sector, whose GVA is projected to fall by around 1.4%. Again, this result is quite in line with expectations, especially considering the high energy requirements of the manufacturing sector. The Agriculture, forestry and fishing industry is also significantly affected by a drop in manufacturing final demand. This is especially due to the fact that the Agriculture forestry and fishing sector is the single most important supplier to the Manufacturing of food products and beverage sector. At the same time, the latter sector is the single most important source of demand for the production of the Agricultural and fishing sector.

Results vary significantly when induced effects are internalised. The drop in labour demand brought about by both direct and indirect falls in intermediate production, leads to a reduction in labour income and consequently in household consumption. Thus the sectors which will be affected the most after internalising induced effects, will be those sectors which happen to absorb the highest proportion of household consumption final demand. These include the Wholesale and retail trade sector (with a drop of around 2% in its GVA), Accommodation and foods services activities sector (fall of 1.7% in GVA), Real estate activities Sector (-1.7% in GVA) and Other Services activities sector (-1.3% in GVA).

Table 10: Aggregate Results for a 10% shock in the final demand of the Manufacturing sector

<table>
<thead>
<tr>
<th></th>
<th>Simple Direct+Indirect</th>
<th>Total Simple+Induced</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Domestic Product</td>
<td>-1.37</td>
<td>-1.95</td>
</tr>
<tr>
<td>Household consumption</td>
<td>-0.87</td>
<td>-2.00</td>
</tr>
<tr>
<td>Government consumption</td>
<td>-0.17</td>
<td>-0.43</td>
</tr>
<tr>
<td>Gross Fixed Capital Formation</td>
<td>-0.81</td>
<td>-1.18</td>
</tr>
<tr>
<td>Exports</td>
<td>-1.99</td>
<td>-2.16</td>
</tr>
<tr>
<td>Imports</td>
<td>-1.53</td>
<td>-1.90</td>
</tr>
<tr>
<td>Aggregate employment</td>
<td>-1.32</td>
<td>-1.91</td>
</tr>
<tr>
<td>Aggregate labour income</td>
<td>-1.30</td>
<td>-1.80</td>
</tr>
</tbody>
</table>

Results defined as % deviation from baseline levels following a 10% drop in manufacturing final demand.

The general conclusions derived from sectoral results are reflected in the aggregate final demand results shown in table 10. An open Leontief model (that is excluding induced effects), suggests a drop of around 1.4% in overall GDP following a 10% drop in manufacturing final demand. In line with sectoral results which suggest significant drops in the value added of export oriented sectors, aggregate exports are expected to be the main driver behind the fall in aggregate GDP,
with a fall of around 2%. Given the considerable import intensity of final demand components in Malta, falls in aggregate demand are expected to be partially outweighed by a drop in import demand. As is typically seen in Leontief driven models, the fall in the demand for labour input is roughly in line with the drop in GDP, with similar drops registered in aggregate labour income. It is important to note that since the results of the first column of table 10 are based on Simple multipliers, the effects on private consumption of the fall in labour income is not being internalised. Despite failing to internalise induced effects, results based on Simple multipliers still project a fall in household consumption of around 0.9%. Since in this case income effects are not internalised, the vast majority of the drop in household consumption is due to the direct effect of the drop in final demand. Indeed, part of the final demand absorbed by the manufacturing sectors takes the form of private consumption expenditure, which is therefore being implicitly shocked in this simulation.

This fact points at an important limitation of this integrated model. While these two modules allow their user to individually shock the final demand of every sector in the economy, it is unable to distinguish between the different types of final demand. In other words, the model is unable to \textit{endogenously} capture the potentially different effects that say, a shock to the consumption final demand of the Manufacturing sector might have when compared to a shock to the exports final demand of the same sector.\textsuperscript{10}

When we move towards using Total multipliers, that is those consistent with a closed Leontief model which endogenises household consumption behaviour, we see how the projected drop for aggregate household consumption is expected to increase significantly. The further fall in aggregate demand leads to further reductions in Gross Fixed Capital Formation and aggregate labour demand, which also reduces aggregate labour income. The fall in aggregate demand is partially outweighed by further drops in the demand of imported goods and services. Still the drop in GDP is expected to increase to around 2% when internalising induced effects.

Two other important limitations of this link stem from its reliance on the Input Output model to provide all results. Indeed, in line with other IO based models, and contrary to results derived from the EC \textrightarrow IO link, the results derived in this section implicitly assume constant prices.

\textsuperscript{10}To see this more formally, one can note that matrix $S$, in which the user loads the sectoral shocks and that are used in equations 11, 19, 20 and 21, does not have a dimension for different final demand components $k$. The model user can however implicitly internalise any additional information on the composition of the shock in the calibration process of $S$. 
Moreover, for a given set of shocks, the model is only able to capture how a shock disseminates across different sectors but not how it propagates across time. In this light, the results of this link might be interpreted as being valid over the medium-to-long run when shocks would have fully propagated across both industry and time dimensions, and when any price pressures derived by a mismatch between supply and demand would have dissipated (Cassar, 2015).

5.2.2 Shock to manufacturing primary imports

Table 11 provides sectoral GVA results following a 10% drop in one of the primary inputs of the Manufacturing sector. Similar to the case of a manufacturing demand shock, this shock is performed to all nine manufacturing sub-sectors and is calibrated such that the share of primary imports of each sub-sector in total manufacturing primary inputs remains constant. This shock is performed within the Ghoshian module of this integrated model. Results therefore do not only capture the direct effects of a drop in primary inputs of the manufacturing sector, but also the indirect effects, which in this module are measured in terms of forward linkages.

As expected, the sector that is projected to be hit hardest by this supply-side shock is the Manufacturing sector itself, whose GVA is expected to fall by almost 5%. The rest of the sectoral effects are limited to the indirect effects caused by a reduction in manufacturing output. Some of the Manufacturing output forms part of the intermediate inputs used by downstream sectors in their production process. Therefore, a shock which restricts manufacturing output will also indirectly affect, in a negative way, the production process, and consequently the GVA, of the other sectors which use this output as an intermediate input.

The sectors that are mostly affected by indirect effects are: (i) Agriculture, forestry and fishing; (ii) Mining, quarrying and construction; and (iii) Accommodation and food services activities. These capture the interlinkages between the various sectors. For instance, the sub-sectors covering manufacturing of food products and repair and installation of machinery and equipment are very important suppliers of intermediate production to the agriculture sector. Similarly, the mining, quarrying and construction sector absorbs a considerable proportion of the output

11When looking at the model documentation in section 4.3, one notes that all equations in the IO → EC link have a time subscript. It is important to note that the model will only produce time dynamic results if the user has a-priori information on how the exogenous shock is likely to move across time. With such information, the modeller can input different sectoral shocks for each time period in the different columns of matrix $S$ in equation 11.
produced in the Manufacture of other non-metallic mineral products and the Manufacture of fabricated metal products sectors. Finally, results for the Accommodation and food services sector are driven by the fact that this sector, through direct and indirect production rounds, absorbs almost a quarter of all the output of the Manufacturing of food, products, beverages and tobacco. Still, when considering the magnitude of the shock, one can conclude that the responses of the sectors not directly hit by import restrictions are relatively small. This is mainly due to the fact that the sectors being subject to the initial shock are mainly export-oriented, with a limited contribution to the intermediate production process of the rest of the economy.

**Table 11:** Sectoral Gross Value Added Results for a 10% drop in primary imports of the manufacturing sector

| 01-03 Agriculture, forestry and fishing | -0.79 |
| B05-09 F41-43 Mining, quarrying and construction | -0.71 |
| C10-33 Manufacturing | -4.97 |
| D35E36-39 Electricity, gas, steam... | -0.08 |
| G45-47 Wholesale and retail trade... | -0.13 |
| H49-53 Transportation and storage | -0.12 |
| I55-56 Accommodation, food services activities... | -0.53 |
| J58-63 Information and communication | -0.12 |
| K64-66 Financial and insurance activities | -0.02 |
| L68 Real estate activities | -0.10 |
| M69-75 Professional, scientific and technical... | -0.15 |
| N77-82 Administrative and support services | -0.10 |
| O84 Public administration and defence | -0.09 |
| P85 Education | -0.05 |
| Q86-88 Human health and social work activities | -0.17 |
| R90-93 Arts, entertainment and recreation | -0.02 |
| S94-96 Other service activities | -0.31 |
| T97-98U99 Households as employers... | 0.00 |

Results defined as % deviation from baseline levels following a 10% drop in manufacturing primary imports.

This point is reflected in the aggregate final demand results shown in table 12. Indeed, excluding inventories, which as expected fall considerably as producers of manufactured goods run down their existing stocks in the light of import restrictions, the largest declines in aggregate final
Table 12: Aggregate Results for a 10% drop in primary imports of the manufacturing sector

<table>
<thead>
<tr>
<th></th>
<th>Direct+Indirect Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Domestic Product</td>
<td>-0.79</td>
</tr>
<tr>
<td>Household consumption</td>
<td>-0.52</td>
</tr>
<tr>
<td>Government consumption</td>
<td>-0.16</td>
</tr>
<tr>
<td>Gross Fixed Capital Formation</td>
<td>-0.73</td>
</tr>
<tr>
<td>Inventories</td>
<td>-2.26</td>
</tr>
<tr>
<td>Exports</td>
<td>-1.06</td>
</tr>
<tr>
<td>Imports</td>
<td>-0.86</td>
</tr>
</tbody>
</table>

Results defined as % deviation from baseline levels following a 10% drop in manufacturing primary imports.

demand components are seen in total exports. The latter are expected to fall by more than 1%, with the main driver being the Manufacture of computer, electronic and optical products. Gross fixed capital formation is expected to fall by more than 0.7%, mainly on the back of a reduction in the output of the Mining, quarrying and construction sector. Since a considerable proportion of the output of the Manufacturing of food products and the Accommodation and food services sectors are directly consumed by households, falls in the production capabilities of these two sectors brings about a fall in aggregate household consumption of around 0.5%.

This result points at an important limitation associated with this particular module. Indeed, the Ghoshian model assumes that sectors, or final users of these sectors, are unable to substitute any inputs (be it primary or intermediate inputs, or indeed final production) with supplies that are either produced by other sectors or imported from abroad. Thus, in this case, when faced by a fall in the supply of manufactured food products and of accommodation and food services, households or intermediate sectors, are assumed not to be able to substitute this shortfall in supply with imported alternatives.

Moreover, similar to the Leontief model with respect to a final demand shock, a Ghoshian model is unable to capture the potentially different effects of shocks to different types of primary inputs. This means that the results of a relative shock (calibrated as a share rather than in millions) to imports, will only differ from a similarly calibrated shock to, say, labour income, by the extent of the difference in the contribution these two primary inputs have in the total primary inputs of each sector. Moreover, the model cannot perform shocks to the total imports used in each
sector, but is limited to capture shocks to imports that are used in the intermediate production process. This therefore excludes sectoral imports that are directly associated with final demand.

6 Conclusion

This paper documents a sectoral extension to STREAM, the Central Bank of Malta’s main macroeconometric model. The extension proposed in this paper is similar to other integrated models which are prevalent in regional economics literature. The integration strategy utilised in this model, is of a linking type, implying that there is a very clear and definite order of recursion between the different modules that make up the integrated model. Unlike other integrated models commonly found in literature, the model proposed here utilises three different modules that allow for slightly different integration regimes. This means that the model can be put to three different uses depending on the type of questions the researcher might have in mind.

First, this integrated model can be used to find a sectoral decomposition of the responses following a set of aggregate shocks which are consistent with the aggregate results of the error-correction component. This is achieved through an EC → IO link in which the order of recursion flows from the error-correction module to the input-output modules. This link is quite useful whenever the researcher is aware of aggregate shocks and wishes to estimate a sectoral decomposition of model consistent responses. The model also utilises two different input-output modules, a demand and a supply driven Input Output module, which are quite useful whenever the researcher possesses sectoral information on sector-specific demand or supply-side shocks and wishes to understand how these sectoral shocks can affect the other sectors in the economy as well as how these might impact developments in GDP and in its aggregate components. This part of the model can be quite useful whenever the forecaster wishes to internalise sector-specific information within the forecasts in a more complete and transparent way.

This paper also documents a number of simulations that make use of all three modules and which should help the reader understand the different uses of this integrated model. The first two shocks, which are of an aggregate nature, illustrate the functioning of the EC → IO link. As expected a-priori, an aggregate foreign demand shock is most likely to affect export oriented industries such as the Arts, entertainment and recreation sector and the Financial and Insurance activities. As the external shock propagates within the domestic economy, locally oriented sectors, most notably those whose final demand component is especially dependent on household
consumption, start to be considerably affected by the shock. A household preference shock yields considerably different conclusions from those derived after a foreign demand shock, with strong results registered in the Wholesale and retail sector. Through relatively strong backward linkages, the Electricity generation sector is also considerably affected by this shock.

The last two simulations documented in this paper, show how a shock to a specific sector propagates to the other sectors in the economy, either through backward or forward linkages, depending on the nature and type of shocks considered. A shock to the final demand of the Manufacturing sector, is expected to have considerable direct effects on the same sector. Strong backward linkages are expected to contribute to a considerable drop in Electricity generation GVA. After endogenising household consumption behaviour, results indicate that the sectors which happen to absorb the highest proportion of household consumption final demand will be considerably affected by this shock. These include the Wholesale and retail trade sector and the Accommodation and foods services activities. These results are then reflected in deviations in aggregate demand components, with exports being affected the most when excluding induced effects. Endogenising household consumption behaviour leads to a stronger response in household consumption and GFCF. A shock to the primary inputs of the manufacturing sector is projected to have considerable indirect effects on the Agriculture, forestry and fishing, Mining, quarrying and construction and Accommodation and food services sectors. Strong direct effects on the manufacturing sector are expected to significantly drive down inventories and total exports.

Results derived from this integrated model need to be interpreted with caution as they are deeply sensitive to the assumptions underlying the linking strategy employed. First and foremost, results are very much reliant on the data within the Input Output table at the heart of the IO modules. As is customary in input output literature, this data is updated with a considerable time lag. For an economy which is undergoing deep structural transformations, such as in the Maltese case, this is an especially important limitation. Indeed, the technical coefficients implied by a five-year old input output table might not reflect in a satisfactory way the true structure of a very dynamic economy. The lag in the publication of the input output tables, is very much an issue in the EC → IO link, which depends on the computation of the H matrix (whose elements are derived as the proportion of each final demand component that is demanded in each industry). As argued in section 5.1.1, the sectoral results are very much dependent on the exposure that each of the sector has to a particular final demand component, which in practice is provided by matrix $H$. Therefore, an outdated input output table might not only contain outdated technical
coefficients but also provide an outdated disaggregation of the shock matrix $S$.

The limitations to the sectoral modules are obviously very similar to the limitations of input output modelling in general. First of all, these two parts of the model are unable to distinguish between the different effects of different final demand or primary input shocks. Moreover, since this part of the model makes an exclusive use of input output based modules, it is unable to capture how the propagation of shocks across sectors changes across time. Despite these limitations, this new sectoral extension to the Bank’s semi-structural model, greatly expands the policy questions that can be answered by researchers and allows for a better integration of sectoral information within the aggregate forecasting process of the Bank.
References


