MEDSEA-FIN: an estimated DSGE model with housing and financial frictions for Malta*

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Abstract

This paper uses Bayesian techniques and Maltese data over the period 2001–2019 to estimate the parameters of MEDSEA-FIN, one of the Central Bank of Malta’s DSGE models. The model captures linkages between the housing sector, banks and the rest of the economy via a borrowing collateral constraint. The paper shows that the data is informative on a subset of the parameters, and documents that the dynamic properties of the estimated model are in line with similar DSGE models estimated for other countries. The results corroborate recent empirical findings for Malta documented in other studies. The model is used to decompose recent macroeconomic data and shows that housing demand shocks were important drivers of house prices and credit. Shocks from the euro area also drove a significant share of macroeconomic fluctuations. The paper also shows that the model survives external validation tests. Although the model remains somewhat stylized along some dimensions, estimation makes it suitable for policy analysis related to housing and credit markets and associated macroprudential policies.

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1 Introduction

The Central Bank of Malta has a suite of structural, semi-structural and reduced form models that it uses for forecasting and policy analysis. Dynamic Stochastic General Equilibrium (DSGE) models nowadays play a central role in the analysis of policy despite their limitations (Christiano et al., 2018; Gürkaynak and Tille, 2017), primarily because of their theoretical underpinnings and microfounded structure which allows for structural analysis. Consequently, they also form part of the Bank’s policy toolkit. MEDSEA-FIN (Gatt et al., 2020) is one of the DSGE models that have been developed for policy analysis. It was developed with the primary aim to capture linkages between the real estate sector, banks and the macroeconomy, and therefore study the role that macroprudential policy tools, particularly loan-to-value and bank capital to asset ratio limits, can play. The Macroprudential Policy of the Central Bank of Malta is governed by Article 136(7) of EU Directive 2013/36/EU, transposed in Central Bank of Malta Directive No.11 and implemented as from 2016 in the case of the counter-cyclical capital buffer and 2019 in the case of borrower-based restrictions.

The model has a rich structure, featuring credit-constrained households, multiple savings vehicles including bank deposits, foreign bonds, capital and housing, a segmented labour market, a domestic non-tradable goods sector, an export sector, a construction sector, banks and a macroprudential authority. This makes the model highly policy relevant, ideal for the study of issues related to the real estate sector, household leverage and the conduct of macroprudential policies. The model as documented in Gatt et al. (2020) is entirely calibrated. Although calibrated models can be useful in the analysis of policies, they are a source of uncertainty with respect to the size and persistence of shocks, the extent of internal endogenous propagation that feeds off these shocks, and their relative contribution to fluctuations of the data. Consequently, it is imperative that the behaviour of the model is disciplined by the data. Nevertheless, issues such as short time series, generally volatile dynamics and structural breaks in the data make estimating such models somewhat challenging. These issues are common when estimating DSGE models for small open economies which have particularly volatile data (Adolfson et al., 2013).

This paper estimates a subset of the model’s parameters using Bayesian methods as discussed in An and Schorfheide (2007) and Fernández-Villaverde (2010). It then backs out the history of structural shocks and studies their contribution to cyclical fluctuations in Malta over the past two decades. This is the first medium-scale DSGE model to be estimated using a rich set of observables for the Maltese economy. Besides providing estimates for a subset of the model’s parameters, the paper studies the implied dynamics of the model to a number of structural shocks, and decomposes recent economic history into its structural drivers.

The data is informative on several of the model’s parameters, and the model generates impulse responses to structural shocks that are in line with expectations and those from other estimated DSGE models. The estimated model predicts a tight link between developments in export price competitiveness, exports and real output, given the large export to GDP ratio in Malta. Historically, housing demand shocks explained a significant share of movements in house prices and credit over the past twenty years, and some of this fed through to consumption via the

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collateral effect of household borrowing. Foreign (euro area) shocks also contributed to a sizeable share of the cyclical movements in prices and output, again reflecting the nature of a small open economy. Some of these results complement the empirical approaches discussed in Gatt and Ruisi (2020) and Gatt and Ruisi (2022). In fact, a subset of the structural shocks from MEDSEA-FIN overlap strongly with counterparts identified in Structural Vector Autoregressions (SVARs). Moreover, the model survives an external validity test, as it can reasonably reproduce the path for exports even though this data is never used in the estimation. Finally, the paper shows that estimating with data that covers the COVID-19 pandemic leads to a sizeable shift in some parameter posterior distributions, as the model tries to explain the large fluctuations using a few of the structural shocks.

This paper therefore contributes to the literature on estimated DSGE models with financial frictions (Gerali et al., 2010; Funke et al., 2018), particularly medium-scale models used for policy analysis (Adolfson et al., 2013; Del Negro et al., 2013; Coenen et al., 2018), and yields an estimated DSGE model for Malta that can be used for policy analysis related to housing and macroprudential policy.

The rest of this paper is structured as follows. Section 2 provides a bird’s eye view of the model and documents some modifications to the calibrated version documented in Gatt et al. (2020). Section 3 then discusses the data used, the Bayesian estimation approach used, including the priors and the estimated parameter posterior distributions. Section 4 shows the properties of the estimated model using impulse response function analysis and variance decompositions, while Section 5 re-estimates the model using data that covers the pandemic period too. Section 6 summarizes these findings and proposes avenues for future research.

2 The model

2.1 MEDSEA-FIN at a glance

MEDSEA-FIN is a multi-sector, medium-scale open-economy DSGE model featuring nominal, real and financial frictions. It is based on the Two-Agent New Keynesian (TANK) model framework (Bibbii, 2008; Campbell and Mankiw, 1989; Debortoli and Gali, 2017; Gali et al., 2007) that imposes limited household heterogeneity, distinguishing between patient and unconstrained households (savers), and impatient and constrained households (borrowers). The presence of limited enforcement on debt repayment means that borrowers face a collateral constraint that is binding, with a borrowing limit that fluctuates with house prices over the business cycle. The limited household heterogeneity is sufficient to capture first order effects such as amplification due to financial frictions and stabilization from macroprudential policy. Households supply a differentiated labour service and therefore exercise some degree of monopoly power over the real wage rate $w_{m,i,t}$ for labour hours worked by household $i$ in sector $m$, that are provided to a labour packer.

The model also features a real estate sector, a banking sector and a rich production environment with local intermediate and final goods producers, importers and exporters, and an exogenous sector representing the rest of the world. It distinguishes between three main production sectors, producing intermediate non-tradable (NT) goods ($Y^{NT}_i$), intermediate export goods...
goods \((Y_{t}^{XD})\) and domestic housing construction \((Y_{t}^{H})\). All production sectors use labour and capital as factors of production. Capital used in the domestic non-traded and housing sectors is accumulated through investment by saver households, while capital used to produce the export good is determined exogenously, to reflect the reality that in very small and open economies investment in the tradeable sector is largely determined by foreign direct investment. Importers buy a homogenous good from the foreign economy, re-brand it and sell it at as \(Y_{t}^{M}\) (after applying a mark-up) to final goods producers. The latter use a mix of this imported good and the non-traded good \((Y_{t}^{NT})\) to produce final consumption \((Y_{t}^{C} = C_{t})\) and investment \((Y_{t}^{I} = I_{t})\) goods. Exporters combine imports \((Y_{t}^{M})\) and the domestically produced export good \((Y_{t}^{XD})\) to produce the final export good \((Y_{t}^{X})\).

Banks intermediate resources between savers and borrowers. Both households and banks are subject to regulatory restrictions that take the form of limits on household LTV and bank capital-to-asset ratios respectively. A macroprudential authority, driven by financial stability objectives, uses these limits as policy tools and changes them over the financial cycle to exert some influence on the economy. The authority systematically tightens (loosens) these requirements when the credit-to-GDP ratio is above (below) trend. The government consumes the non-traded good and balances the budget in each period by levying non-distortionary lump sum taxes on households.

I make a few changes to the model from its documented version (Gatt et al., 2020). These are changes to the dynamic structure of leverage limits for both households and banks, the introduction of mark-up shocks, the modification of the foreign block and adjustments to some market clearing conditions. I discuss these in more detail below.

The model economy is perturbed by 16 structural shocks; an intertemporal preference shock \(\epsilon_{t}^{\beta}\), a housing preference shock \(\epsilon_{t}^{H}\), a labour supply shock \(\epsilon_{t}^{N}\), a credit supply shock \(\epsilon_{t}^{m}\), productivity shocks in the intermediate non-traded goods \(A_{t}^{NT}\), intermediate export goods \(A_{t}^{XD}\) and housing \(A_{t}^{H}\) sectors respectively, mark-up shocks in the intermediate non-traded goods \(\mu_{t}^{NT}\), intermediate export goods \(\mu_{t}^{XD}\) and the intermediate import goods \(\mu_{t}^{M}\) sectors, a government spending shock \(g_{t}\), shocks to foreign output \(\epsilon_{t}^{Y^{\ast}}\) and prices \(\epsilon_{t}^{P^{\ast}}\), a foreign monetary policy shock \(\nu_{t}^{R^{\ast}}\), an external risk premium shock \(\epsilon_{t}^{\phi}\) and a foreign direct investment shock \(K_{t}^{XD}\). The labour supply, risk premium, export price markup and foreign direct investment shocks proved difficult to identify and estimate given the available data, so I therefore shut them off during estimation. I also shut off the government spending shock, since the model assumes a simple government consumption rule which delivers counterintuitive results in general equilibrium. This leaves 11 structural shocks to be estimated. All the equations are reported in Appendix A.

### 2.2 Modifications to the model

#### 2.2.1 Notation

I modify the notation on some parameters slightly to make it consistent and easier to read the estimation results. For example, the adjustment cost parameter for the price of the non-traded

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2These shocks are not presented in the documented version but are discussed further below.

3I also change the government spending equation such that it does not react to shocks.
good $Y^\text{NT}_t$ is $\xi^\text{NT}_P$, while that for investment in capital used to produce the non-traded good $I^\text{NT}_t$ is $\xi^\text{NT}_I$. This notation clarifies that the first relates to rigidity in prices (subscript $P$) in the NT sector while the second relates to investment rigidity (subscript $I$) in the same sector. Similar changes are made for other adjustment cost as well as price indexation parameters.\footnote{I also take note and correct a few typos in the original paper. In the budget constraint for patient households the price of investment goods across sectors should be different rather than common at $p^I_t$. The first order conditions show the correct different prices. In the bank problem, the flow of funds constraint is written in real terms, but the FOCs are expressed in nominal terms. Moreover, the adjustment costs had the wrong sign in the budget constraint. These errors were present in the paper but not in the code that generates the results shown therein.}

Since the numeraire in MEDSEA-FIN is the price of the foreign economy, I use lowercase to denote prices relative to the consumption deflator (for example, $p^H \equiv P^H / P^C$).

### 2.2.2 Re-calibration

I make a few changes to a few of the calibrated parameters of the model. I re-calibrate the utility weight on housing, which pins down the steady state ratio of housing wealth to output. In the calibrated version of the model this ratio (4.55) overshoots the target in the data of 4. Since housing contributes to the key dynamics of interest in this model, I re-calibrate the value of $\hat{\epsilon}^H$ such that the target is met exactly. This comes at the cost of lowering the steady state fraction of domestic non-tradable goods prices in the final export price, governed by $\theta$, from 13.8% to 10%.$^5$ In preliminary results the posterior medians for the Rotemberg wage adjustment costs were low, which led to wages reacting too strongly to shocks. Furthermore, real rigidity in the model implied that even prices in Malta adjust too strongly following a shock. To reduce this, I fix the Rotemberg wage adjustment cost parameter at 200 for all three sectors and I increase the substitutability between labour types in the utility functions of households by lowering the elasticity parameter $\varsigma$ from 1 to 0.5.

The second change to the calibration concerns the macroprudential policy response to fluctuations in credit. These policy tools (LTV and bank capital to asset ratio limits) were not in use throughout most of the sample used in the estimation. Therefore, I switch off the policy rules, such that the regulatory capital-to-assets and LTV ratios are always constant in the estimation period.

Since in preliminary estimations some parameters were not identified, I fix them at their calibrated value. Additionally, since I do not use labour market data in the set of observables, the parameters relating to wage setting are unidentified. Moreover, the model exhibited relatively weak nominal rigidity in the wake of shocks, so I increase the value of the Rotemberg wage adjustment cost parameter $\xi^W_j, j \in \{\text{NT, XD, H}\}$ to 200 to achieve more reasonable general equilibrium responses.

### 2.2.3 Sluggish household borrowing and credit shocks

In the original model impatient households face a collateral constraint that binds in every period to a fraction of the expected value of housing. When binding, this constraint forces borrowing

\footnote{In any case, this target is set somewhat arbitrarily in the original calibration, given the absence of data on the share of Maltese export prices accounted for by the price of domestic non-tradable goods.}
in every period to be revised in line with movements in house prices, the LTV ratio, the stock of housing held and goods price inflation. To reflect the slow-moving nature of the stock of credit over time in the data, I follow Iacoviello (2015) and Guerrieri and Iacoviello (2017) and introduce sluggish changes to the borrowing limit as:

\[ L_{b,t} \leq \rho_L \frac{L_{b,t-1}}{\Pi_t^L} + (1 - \rho_L) \Pi_t^C \mathbb{E}_t \left\{ \frac{p_{H,t+1} H_{b,t} \Pi_t^C}{R_t^L} \right\} \]  

(1)

where \( \rho_L \) denotes the degree of inertia. Note that the LTV ratio \( m \) is written as a constant. I also introduce a shock to borrowing conditions \( \varepsilon^m_t \) as in Darracq Pariès and Notarpietro (2008) and Iacoviello (2015), which captures bank-level changes in lending policies and other factors that can make new credit issuance deviate from the borrowing limit. This credit shock follows an AR(1) process in logs:

\[ \log(\varepsilon^m_t) = \rho_m \log(\varepsilon^m_{t-1}) + \nu^m_t. \]  

(2)

These modifications imply a few changes to the optimality conditions for consumption smoothing and housing demand as follows:

\[ \lambda_{b,t} = \beta_b \mathbb{E}_t \left\{ \frac{R_t^L \lambda_{b,t+1} - \rho_L \lambda_{t+1}^L}{\Pi_{t+1}^L} \right\} + \lambda_t \]  

(3)

\[ \frac{\varepsilon^H_t}{\Pi_t} = p^H_t \lambda_{b,t} - \beta_b (1 - \delta_H) \mathbb{E}_t \left\{ \frac{p^H_{t+1} \lambda_{b,t+1}}{\Pi_{t+1}^L} \right\} - (1 - \rho_L) \lambda_{b,t} m \varepsilon_t^m \mathbb{E}_t \left\{ \frac{p_{H,t+1}^L \Pi_{t+1}^C}{R_t^L} \right\}. \]  

(4)

This modification yields debt dynamics that are slower and persistent, which improves the fit of the model and is a short-hand way of introducing long term debt as in Justiniano et al. (2015), Chen and Columba (2016), Gelain et al. (2018) and Grodecka (2020). This specification also nests the constraint in the documented version when \( \rho_L = 0 \), so the sluggish adjustment is not necessarily imposed on the data.

### 2.2.4 Mark-up shocks

The documented version of MEDSEA-FIN abstracts from mark-up shocks. In the estimated version I add mark-up shocks to the intermediate non-tradable, import and export goods sectors as shocks that perturb the elasticity of substitution around its steady state value. The time-varying mark-up in sector \( m \) is defined as \( \mu^m_t / (\mu^m_t - 1) \). A negative shock to the elasticity of substitution reflects a drop in competition and therefore increases mark-ups. Therefore, the laws of motion for the elasticities are:

\[ \log(\mu^m_t) = \rho_{\mu,m} \log(\mu^m_{t-1}) + (1 - \rho_{\mu,m}) \log(\overline{\mu^m}) - \nu^m_t \]  

(5)

\( \forall m \in \{NT, M, XD\} \), where \( \overline{\mu^m} \) is the steady-state value of the elasticity. These shocks yield a structural interpretation of observed price changes and improve the fit of the model. The steady state values are kept the same as in Gatt et al. (2020). Note that a positive innovation to \( \nu^m_t \) lowers the elasticity, therefore increasing the mark-up.
2.2.5 The foreign block

In the original model the variables that make up the foreign (euro area) block are not described by micro-founded optimality conditions. Instead, euro area output $Y^*$ and prices $P^*$ are specified as AR(1) processes in logs, while the interest rate $R^*$ is constant. Although parsimonious, this specification misses the link between prices and economic activity in the euro area and therefore abstracts from the importance of monetary policy and its effects on euro area output and prices. It also results in euro area shocks that transmit solely through a rise in Maltese exports, which puts upward pressure on domestic prices, without an accompanying rise in euro area prices. This leads to unfavourable (and implausible) terms of trade movements. Moreover, the simple univariate structure also misses the transmission of monetary policy to the Maltese economy.

To introduce some economic structure into this block, I re-specify the equations for $Y^*$ and $P^*$ as semi-structural IS and Phillips Curve relations respectively. Output relative to steady state values, with response parameters $1 + \tau_{Y^*}$ and $\tau_{Y^*}$ respectively. The interest rate response displays inertia as is common in the literature, governed by the parameter $\rho^*$. Prices are also persistent around their steady state value (normalised to 1) with an autoregressive parameter $\rho^*_Y$, and move pro-cyclically with output with sensitivity $\phi_{Y^*}$. This mimics the relation between marginal costs and price pressures that results from price setting frictions in microfounded New Keynesian models. To close the model, I write a Taylor rule for $R^*$ that is a function of the deviation of foreign inflation and gross output growth from their steady state values, with response parameters $1 + \tau_{Y^*}$ and $\tau_{Y^*}$. The interest rate response parameters $1 + \tau_{Y^*} = 1 + 0.1$ and $\tau_{Y^*} = 1 + 0.1$. The interest rate response parameters $1 + \tau_{Y^*} = 1 + 0.1$ and $\tau_{Y^*} = 1 + 0.1$. The shocks $\varepsilon_t^{Y^*}$, $\varepsilon_t^{P^*}$ and $\nu_t^{P^*}$ are interpreted as euro area demand, supply and monetary policy shocks respectively. This set-up ensures the small-country assumption for Malta such that euro area variables do not react to Maltese variables, and is similar to that specified in Výškrabka et al. (2019). In the estimation I use the Short-term Shadow Rate (SSR) (Krippner, 2020) as the observed policy rate. This has the benefit of capturing both conventional and unconventional monetary policy shocks and their effect on

\[
\dot{Y}_t^* = \phi_{Y^*} Y_t^* - \phi_{2} Y_t^* R_t^* - \varepsilon_t^{Y^*} (1 - \phi_{1} Y_t^*)
\]

\[
\log(P_t^*) = \rho_{P^*} \log(P_{t-1}^*) + \phi_{P^*} \log(Y_t^*) + \varepsilon_t^{P^*}
\]

\[
R_t^{a,s} = \rho_{R^*} R_{t-1}^{a,s} + (1 - \rho_{R^*}) \left( \bar{R}_t^{a,s} + (1 + \tau_{Y^*}) \pi_t^{a,s} + \tau_{Y^*} \left( \frac{Y_t^*}{Y_{t-1}^*} - 1 \right) \right) + \nu_t^{P^*}
\]

where $\varepsilon_t^{Y^*} = \rho_{Y^*} \varepsilon_{t-1}^{Y^*} + \nu_t^{Y^*}$, $\varepsilon_t^{P^*} = \rho_{P^*} \varepsilon_{t-1}^{P^*} + \nu_t^{P^*}$, $\bar{R}_t^{a,s} = (1/\beta_s - 1) 400$, $\Pi_t^* = P_t^* / P_{t-1}^*$ and $\pi_t^{a,s} = (P_t^* / P_{t-4}^* - 1) 100$. The shocks $\varepsilon_t^{Y^*}$, $\varepsilon_t^{P^*}$ and $\nu_t^{P^*}$ are interpreted as euro area demand, supply and monetary policy shocks respectively. This set-up ensures the small-country assumption for Malta such that euro area variables do not react to Maltese variables, and is similar to that specified in Výškrabka et al. (2019). In the estimation I use the Short-term Shadow Rate (SSR) (Krippner, 2020) as the observed policy rate. This has the benefit of capturing both conventional and unconventional monetary policy shocks and their effect on

\footnote{The specification of the response parameter on inflation enforces the Taylor Principle in the estimation when the prior and posterior mass for $\tau_{Y^*}$ is restricted over a positive domain.}

\footnote{These modifications affect the dynamics of the euro area system but not its steady state, and therefore it does not affect the steady state of the full model.}
euro area and Maltese macroeconomic variables. It also helps me side-step the non-linearity associated with the zero lower bound on the actual policy rate. Since the parameters in equations (6), (7) and (8) are largely in reduced form I calibrate their values such that they generate dynamic responses in output and inflation to a monetary policy shock that closely follow those in the New Area Wide Model II (NAWM-II) (Coenen et al., 2018). I achieve this by setting $\phi_Y^* = 0.36$, $\phi_{Y^*}^2 = 1.51$, $\rho_{P^*} = 0.9$, $\phi_{P^*} = 0.12$, $\rho_{R^*} = 0.85$, $\tau_{Y^*} = 1.19$, and $\tau_{Y^*} = 0.1$.

2.2.6 Nominal and real GDP measurement

I change the market clearing condition that defines real output from the production side to account for the fact that its elements are measured in different prices. This issue is rarely discussed in the literature, and when it is, it is typically relegated to a footnote or appendix. Following Faruqee et al. (2007), I define real output $Y_t$ along the ‘national accounts’ statistical concept: output at constant market prices. Therefore, I define nominal output as output at current market prices, and real output at fixed steady state prices. The deflator $P_Y$ then tracks the evolution between these two measures when the model is away from the steady state. The new specification is given by:

\[ P_Y Y_t = P_Y^C C_t + P_Y^I I_t + P_Y^{NT} Y_t^{G} + P_Y^X Y_t^X - P_Y^M Y_t^M \]  

\[ Y_t = P_Y^C C_t + P_Y^I I_t + P_Y^{NT} Y_t^{G} + P_Y^X Y_t^X - P_Y^M Y_t^M \]

which implicitly defines $P_Y = 1$ in the steady state.

3 Estimation details

Let $s_t$ denote the $N_s \times 1$ vector of the states of the model, and $y_t$ the $N_y \times 1$ vector of observed variables; then the linearised rational expectations solution is given by the state space representation:

\[ s_t = As_{t-1} + \nu_t, \quad \nu_t \sim N(0, Q) \]  

\[ y_t = Bs_t + \varrho_t, \quad \varrho_t \sim N(0, R) \]

where $\nu_t$ is a $N_\nu \times 1$ vector of structural shocks with time-invariant variance-covariance matrix $Q$ of size $N_\nu \times N_\nu$, and $\varrho_t$ an $N_\varrho \times 1$ vector of shocks that capture both measurement errors as well as the possibility that the mapping from the model to the data is not perfect. The time-invariant matrix $R$ is of size $N_\varrho \times N_\varrho$, and both variance-covariance matrices are assumed to be diagonal. Equation (11) is the transition equation and (12) the observation/measurement equation. The elements of $A$ are functions of the structural parameters, while the elements of $B$ map the model states to the observables. A necessary requirement for likelihood-based estimation of the model’s structural parameters is that $N_\nu + M_\varrho \geq N_y$, otherwise the system is singular.

In a Bayesian context the objective is to find the posterior distribution of the structural parameters $\theta$ given the data, $p(\theta|\mathcal{Y})$, where $\mathcal{Y} = \{y\}_1^T$. Using Bayes’ rule, the posterior is a
function of the likelihood \( L(\theta | Y) = p(Y | \theta) \), the prior \( p(\theta) \) and the marginal data density \( p(Y) \):

\[
p(\theta | Y) = \frac{L(\theta | Y)p(\theta)}{p(Y)}.
\]

For computational purposes it is common to work with the log posterior kernel, which is proportional to the log-likelihood and the log prior:

\[
\log p(\theta | Y) \propto \log p(\theta) + \log L(\theta | Y).
\]

The log-likelihood function is:

\[
\ln L(\theta | Y) = -\frac{T N_y}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^{T} \ln |F_t| - \frac{1}{2} \sum_{t=1}^{T} \eta_t' F_t^{-1} \eta_t
\]

with \( \eta_t = y_t - Bs_{t|t-1} \) as the one-step prediction error and \( F_t = BP_{t|t-1}B' + R \) the associated covariance matrix, where \( P_{t|t-1} = \mathbb{E}(s_{t-1} - s_{t|t-1})(s_{t-1} - s_{t|t-1})' \) (Harvey, 1990).

### 3.1 Observables

A period in the model is calibrated to be a quarter, and so the data I use in the estimation is also at this frequency. When this is available at different frequencies I apply suitable frequency transformation. For real variables, I follow the approach in Gerali et al. (2010) and Iacoviello and Neri (2010) and map real variables in the model to the actual chain-linked data in per capita terms. I use 11 observables in the benchmark estimation for with a sample period running from 2001Q1 to 2019Q4. For the Maltese block, I use real consumption per capita \( (C) \), real house prices \( (p^H) \), the HICP index \( (P^C) \), the services HICP index \( (P^{Serv}) \), total credit to households per capita \( (L) \), investment in dwellings per capita \( (I^H) \), the import deflator \( (P^M) \) and real GDP per capita \( (Y) \). For the euro area block I use euro area GDP \( (Y^{EA}) \), euro area HICP \( (P^{EA}) \) and the Short Shadow Rate estimate of the ECB policy rate \( (R^{EA}) \) that is based on the methodology of Krippner (2013). Appendix B provides further details on the variables and their sources.

Some labour market variables that can be mapped to the model variables are highly volatile in Malta, as are the data for total investment and foreign direct investment. In preliminary estimations these data proved to be largely uninformative, and I therefore do not use labour market and investment data in the benchmark estimation. I also attempted to include real imports and real exports. However, these variables were a source of large measurement errors when made part of the full set of observables, as the model struggled to make sense of all the mapped data and its dynamics. This is likely due to the absence of additional structural shocks that can account for the historical fluctuations. I therefore do not include them in the final vector.

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8It is hard to obtain consistent series at a quarterly frequency for periods before 2000 for Maltese data, in part due to methodological changes in statistical data collection. Even if all the time series are available, they are unlikely to be informative given the structural changes that the economy experienced in the 1990s, and which are not explicitly modelled.

9The use of variables in per capita terms is especially relevant in the case of Malta, given that it experienced strong population growth during half of the sample period, which averaged 2.5% between 2012 and 2019.
of observables. In any case these variables are unlikely to drive the key channels of interest in the model, as MEDSEA-FIN was built with the aim of studying the role of the real estate market and macroprudential policy, which are driven primarily by several domestic factors. I provide further details on the data and any transformations in Appendix B. Nevertheless, I use data for exports for an ex-post validation exercise in Section 4.3.

I use a difference filter on the data, similar to the approaches adopted by Smets and Wouters (2007), Quint and Rabanal (2014) and Schmitt-Grohé and Uribe (2012). Most data for Malta is not seasonally adjusted by the National Statistics Office, and most policy institutions in Malta typically conduct economic analysis based on year-on-year changes in key macroeconomic aggregates. For these reasons, I work with the yearly growth rate of the variables I use in the estimation. This has the benefit of side-stepping seasonal adjustment techniques. Additionally, the model does not include a balanced growth path and therefore has a stationary steady state. I therefore de-mean the data before mapping it to the variables.

Figure 1 shows the data series used in the estimation. The yearly growth in raw household credit shows a downward trend from the early part of the sample, owing to the financial liberalization that occurred in the mid-1990s, which led to a boom in credit that eventually stabilized. This downward trend is unlikely to be explained by the structural shocks that are in the model. I therefore extract a linear time trend and use the cyclical component around this trend as the data of interest. Furthermore, the data have a statistical break around 2003, which lead to some negative values in the cycle. In preliminary estimations the model had a hard time explaining this negative growth, in a period when house prices where on the rise. I therefore treat this data as latent in this period and let the Kalman filter interpolate this data given all other information from the state space. In Figure 1 I show this filtered series for credit growth. I use the first 4 years of data (2001Q1–2004Q4) as a training sample to initialize the Kalman filter.

\footnote{I also explore the use of total investment and non-dwelling investment but these variables are particularly volatile and interfered with the posterior likelihood maximisation. Treating the data for outliers did not rectify this problem, so I discarded them from the set of observables.}
Figure 1: The data used in the estimation

Note: All the data are de-meaned, and credit growth is detrended.
The system of measurement equations (12) that map the data to the model variables is:

\[ g_{C,t}^{\text{data}} = \left( \frac{C_t}{C_{t-4}} - 1 \right) 100 + \theta_t^{C,\text{ME}} \] (14)

\[ g_{pH,t}^{\text{data}} = \left( \frac{pH_t}{pH_{t-4}} - 1 \right) 100 + \theta_t^{pH,\text{ME}} \] (15)

\[ g_{pC,t}^{\text{data}} = \left( \frac{pC_t}{pC_{t-4}} - 1 \right) 100 + \theta_t^{pC,\text{ME}} \] (16)

\[ g_{L,t}^{\text{data}} = \left( \frac{pC_t L_t}{pC_{t-4} L_{t-4}} - 1 \right) 100 + \theta_t^{L,\text{ME}} \] (17)

\[ g_{tH,t}^{\text{data}} = \left( \frac{Y_{tH}}{Y_{tH_{t-4}} - 1} \right) 100 + \theta_t^{tH,\text{ME}} \] (18)

\[ g_{Y,t}^{\text{data}} = \left( \frac{Y_t}{Y_{t-4}} - 1 \right) 100 + \theta_t^{Y,\text{ME}} \] (19)

\[ g_{pM,t}^{\text{data}} = \left( \frac{pM_t}{pM_{t-4}} - 1 \right) 100 + \theta_t^{pM,\text{ME}} \] (20)

\[ g_{pServ,t}^{\text{data}} = \left( \frac{pNT_t}{pNT_{t-4}} - 1 \right) 100 + \theta_t^{p\text{Serv},\text{ME}} \] (21)

\[ g_{Y\text{EA},t}^{\text{data}} = \left( \frac{Y_{t\text{EA}}}{Y_{t\text{EA}_{t-4}} - 1} \right) 100 \] (22)

\[ g_{P\text{EA},t}^{\text{data}} = \left( \frac{P_{t\text{EA}}}{P_{t\text{EA}_{t-4}} - 1} \right) 100 \] (23)

\[ R_{t\text{EA},t}^{\text{data}} = \left( \frac{R^* - \frac{1}{\beta_x}}{400} \right) \] (24)

where \( \tilde{X}_t = X_t P_{t*}^c, X \in \{ P^C, P^M, P^NT \} \) adjusts for underlying changes in the numeraire. As I note above, I demean the growth rates and interest rate series before mapping them to the model variables. The measurement error for any observable \( Z \) is assumed to follow a Normal i.i.d stochastic process \( \theta_t^Z \sim N(0, \sigma^2_Z) \).

### 3.2 Priors

I estimate 39 parameters and shock variances (including measurement errors), summarized in Table 1, and I set priors on their distributions. The priors on most of these parameters follow the convention in the literature; I use Beta distributions for parameters bound between 0 and 1, Gamma distributions for the adjustment cost parameters and Inverse Gamma distributions for shock variances. Most of the priors are neither loose nor overly tight, and centered around the calibrated values of the parameters. I adopt a more conservative approach for parameters whose values were somewhat arbitrarily calibrated in Gatt et al. (2020), such as for the persistence of shocks, and center the distribution around lower persistence. In preliminary testing the (log) likelihood of the model was flat over wide regions and therefore hard to maximise. I therefore impose some priors more tightly in order to successfully maximise the posterior (log) likelihood.
around which I build the posterior distributions of the parameters. On the other hand, I set the prior mean for the adjustment cost in the accumulation of construction capital \( \xi^H \) to a high number with a relatively wide variance, as I otherwise run into numerical issues with the posterior mode-finding and lack of convergence of the Random Walk Metropolis-Hastings sampler. The priors on the standard deviations of the structural shocks reflect the beliefs that demand shocks (housing and intertemporal preferences) were historically important sources of business cycle fluctuations, followed by technology shocks in the three production sectors. I fix a lower prior on mark-up shocks given that these can account for a substantial share of fluctuations in prices if left loose.

Since Maltese macroeconomic data tends to be particularly volatile and noisy, I calibrate the mean of the prior on the measurement errors such that these account for up to 10% of the variance of the observables (15% for the consumption deflator), similar to the approaches of Adolfson et al. (2013) and Schmitt-Grohé and Uribe (2012). I impose these priors relatively tightly. The priors for parameters that govern the dynamics and shock standard errors are summarized in Tables 2 and 3 below, respectively.

### 3.3 Estimated parameters

I use Dynare version 5.2 (Adjemian et al., 2022) for all estimation and impose bounds on the posterior distributions on some parameters, mainly persistence parameters, to have mass only over the support [0.001 - 0.995] to rule out unit root processes. Figure 2 shows the prior and posterior distributions of the parameters that mainly govern the dynamics of the Maltese block of the model, while Table 2 shows summary statistics. The data is generally informative on most parameters, in particular the persistence of shock processes, although some parameters remain unidentified. The posterior distribution on consumption habits \( \chi \) sits directly on top of the prior, leaving it unidentified. Nevertheless, its mean value is close to the estimate in Coenen et al. (2018) for the euro area. The mean indexation parameters in import and non-tradable price adjustment costs \( \iota^M_{\pi} \) and \( \iota^{NT}_{\pi} \) are both estimated at around 0.41. Indexation to past wage inflation is higher than for prices. These estimates are also in line with the results in Coenen et al. (2018). On the other hand, price stickiness is estimated to be higher in import \( \xi^M_{\pi} \) and export \( \xi^X_{\pi} \) goods than for non-tradable goods \( \xi^{NT}_{\pi} \). The adjustment cost parameter \( \xi^H \) yields high adjustment costs in construction investment.

Turning to the parameters that relate to credit and banking, the estimation returns high inertia in the stock of household loans \( \rho_L \) with a mean of 0.74, close to the 0.70 estimated in Iacoviello (2015) and Guerrieri and Iacoviello (2017). This is evidence that the data clearly reject a household collateral constraint that ties all of outstanding credit volumes to a fraction of housing wealth in every period.

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11See the discussion in Fernández-Villaverde (2010, p. 37) on the role of priors when using models for policy analysis and when models are for ‘pure’ research.

12Some priors with the same mean are imposed more tightly than others due to numerical issues during the maximisation of the posterior mode, but the results shown below are not really sensitive to these assumptions as the data is informative on the parameters relating to the shocks.

13Weak or incomplete parameter identification is a problem that plagues even well-known DSGE models such as the Smets and Wouters (2007) model; see Canova et al. (2014) and the references within.
Table 1: List of the estimated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi$</td>
<td>Household habit parameter</td>
</tr>
<tr>
<td>$\iota^M$</td>
<td>Price indexation: imports</td>
</tr>
<tr>
<td>$\iota^{NT}$</td>
<td>Price indexation: non-tradables</td>
</tr>
<tr>
<td>$\iota^W$</td>
<td>Wage indexation (all sectors)</td>
</tr>
<tr>
<td>$\iota^X$</td>
<td>Price indexation: exports</td>
</tr>
<tr>
<td>$\xi^I$</td>
<td>Adjustment cost: capital in non-tradables</td>
</tr>
<tr>
<td>$\xi^H$</td>
<td>Adjustment cost: capital in housing</td>
</tr>
<tr>
<td>$\xi^{NT}$</td>
<td>Adjustment cost: price setting for imports</td>
</tr>
<tr>
<td>$\xi^{NT}$</td>
<td>Adjustment cost: price setting for non-tradables</td>
</tr>
<tr>
<td>$\xi^{XP}$</td>
<td>Adjustment cost: price setting for exports</td>
</tr>
<tr>
<td>$\rho_L$</td>
<td>Persistence in stock of loans</td>
</tr>
<tr>
<td>$\rho_H$</td>
<td>Persistence in housing demand shock</td>
</tr>
<tr>
<td>$\rho_\beta$</td>
<td>Persistence in discount factor shock</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>Persistence in borrowing conditions shock</td>
</tr>
<tr>
<td>$\rho_{A^{NT}}$</td>
<td>Persistence in technology shock: non-tradables</td>
</tr>
<tr>
<td>$\rho_{A^H}$</td>
<td>Persistence in technology shock: housing</td>
</tr>
<tr>
<td>$\rho_{A^D}$</td>
<td>Persistence in technology shock: exports</td>
</tr>
<tr>
<td>$\rho_{\mu^{NT}}$</td>
<td>Persistence in mark-up shock: non-tradables</td>
</tr>
<tr>
<td>$\rho_{\mu^M}$</td>
<td>Persistence in mark-up shock: imports</td>
</tr>
<tr>
<td>$\rho_{Y^*}$</td>
<td>Persistence of euro area demand shock</td>
</tr>
<tr>
<td>$\rho_{P^*}$</td>
<td>Persistence of euro area supply shock</td>
</tr>
<tr>
<td>$\nu^H$</td>
<td>Shock standard deviation: housing demand</td>
</tr>
<tr>
<td>$\nu^\beta$</td>
<td>Shock standard deviation: discount factor</td>
</tr>
<tr>
<td>$\nu^m$</td>
<td>Shock standard deviation: borrowing conditions</td>
</tr>
<tr>
<td>$\nu^{NT}$</td>
<td>Shock standard deviation: technology in non-tradables</td>
</tr>
<tr>
<td>$\nu^H$</td>
<td>Shock standard deviation: technology in housing</td>
</tr>
<tr>
<td>$\nu^{D}$</td>
<td>Shock standard deviation: technology in exports</td>
</tr>
<tr>
<td>$\nu^\mu^{NT}$</td>
<td>Shock standard deviation: mark-up in non-tradables</td>
</tr>
<tr>
<td>$\nu^\mu^M$</td>
<td>Shock standard deviation: mark-up in imports</td>
</tr>
<tr>
<td>$\nu^{Y^*}$</td>
<td>Shock standard deviation: euro area demand</td>
</tr>
<tr>
<td>$\nu^{H^*}$</td>
<td>Shock standard deviation: euro area supply</td>
</tr>
<tr>
<td>$\nu^{R^*}$</td>
<td>Shock standard deviation: euro area monetary policy</td>
</tr>
<tr>
<td>$\varrho_C$</td>
<td>Measurement error standard deviation: consumption</td>
</tr>
<tr>
<td>$\varrho_H$</td>
<td>Measurement error standard deviation: house prices</td>
</tr>
<tr>
<td>$\varrho_{PC}$</td>
<td>Measurement error standard deviation: consumption deflator</td>
</tr>
<tr>
<td>$\varrho_{PServ}$</td>
<td>Measurement error standard deviation: services deflator</td>
</tr>
<tr>
<td>$\varrho_L$</td>
<td>Measurement error standard deviation: credit to households</td>
</tr>
<tr>
<td>$\varrho_{LH}$</td>
<td>Measurement error standard deviation: investment in dwellings</td>
</tr>
<tr>
<td>$\varrho_{PM}$</td>
<td>Measurement error standard deviation: import deflator</td>
</tr>
<tr>
<td>$\varrho_Y$</td>
<td>Measurement error standard deviation: gross domestic product</td>
</tr>
</tbody>
</table>
Figure 2: Prior and posterior distributions: Dynamics

Notes: The figure shows the prior and posterior distributions as fitted kernel densities from two Random Walk Metropolis-Hastings chains of length 5 million draws each.
Figure 3: Prior and posterior distributions: Shock and measurement error standard deviations

Notes: The figure shows the prior and posterior distributions as fitted kernel densities from two Metropolis-Hastings chains of length 5 million draws each.
Table 2: Prior and posterior distributions: Dynamics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Mean</th>
<th>Std.dev.</th>
<th>Mean</th>
<th>10\textsuperscript{th} Median</th>
<th>90\textsuperscript{th}</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi$</td>
<td>Beta</td>
<td>0.60</td>
<td>0.05</td>
<td>0.60</td>
<td>0.54</td>
<td>0.66</td>
</tr>
<tr>
<td>$\iota_M$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.10</td>
<td>0.40</td>
<td>0.30</td>
<td>0.40</td>
</tr>
<tr>
<td>$\iota_{NT}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.10</td>
<td>0.39</td>
<td>0.27</td>
<td>0.39</td>
</tr>
<tr>
<td>$\iota_W$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.10</td>
<td>0.46</td>
<td>0.34</td>
<td>0.46</td>
</tr>
<tr>
<td>$\iota_P$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.10</td>
<td>0.48</td>
<td>0.36</td>
<td>0.48</td>
</tr>
<tr>
<td>$\xi_H$</td>
<td>Gamma</td>
<td>100.00</td>
<td>50.00</td>
<td>161.53</td>
<td>90.67</td>
<td>153.91</td>
</tr>
<tr>
<td>$\xi_{NT}$</td>
<td>Gamma</td>
<td>20.40</td>
<td>10.00</td>
<td>22.42</td>
<td>11.93</td>
<td>21.19</td>
</tr>
<tr>
<td>$\xi_M$</td>
<td>Gamma</td>
<td>58.00</td>
<td>10.00</td>
<td>49.79</td>
<td>38.41</td>
<td>49.23</td>
</tr>
<tr>
<td>$\xi_{W}$</td>
<td>Gamma</td>
<td>50.00</td>
<td>10.00</td>
<td>41.98</td>
<td>31.29</td>
<td>41.36</td>
</tr>
<tr>
<td>$\rho_L$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.05</td>
<td>0.74</td>
<td>0.69</td>
<td>0.74</td>
</tr>
<tr>
<td>$\rho_H$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.10</td>
<td>0.98</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
<td>$\rho_3$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.49</td>
<td>0.29</td>
<td>0.49</td>
</tr>
<tr>
<td>$\rho_{ANT}$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.10</td>
<td>0.80</td>
<td>0.70</td>
<td>0.81</td>
</tr>
<tr>
<td>$\rho_{AH}$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.10</td>
<td>0.92</td>
<td>0.89</td>
<td>0.92</td>
</tr>
<tr>
<td>$\rho_{AXD}$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.10</td>
<td>0.93</td>
<td>0.90</td>
<td>0.94</td>
</tr>
<tr>
<td>$\rho_{NT}$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.10</td>
<td>0.80</td>
<td>0.66</td>
<td>0.81</td>
</tr>
<tr>
<td>$\rho_{LM}$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.10</td>
<td>0.88</td>
<td>0.80</td>
<td>0.88</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.10</td>
<td>0.94</td>
<td>0.91</td>
<td>0.94</td>
</tr>
<tr>
<td>$\rho_{Y^*}$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.10</td>
<td>0.95</td>
<td>0.91</td>
<td>0.96</td>
</tr>
<tr>
<td>$\rho_{P^*}$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.10</td>
<td>0.36</td>
<td>0.27</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Notes: The last four columns on the right show the estimated mean, median and 10\textsuperscript{th} and 90\textsuperscript{th} percentiles of the posterior distributions. The parameters for the euro area block are estimated separately.

The estimated shock processes are mostly persistent. In particular, the housing preference shock is highly persistent, with a mean value of 0.98 for $\rho_H$. This estimate is in line with the values typically reported in the literature (Iacoviello and Neri, 2010; Iacoviello, 2015). Similarly, the credit supply shock is also highly persistent with a mean of 0.94, as in Durraaq Pariès and Notarpietro (2008). On the other hand, the intertemporal preference shock is much less persistent at a mean value of 0.49, although it appears to be weakly identified. The estimated shock standard deviations are shown in Table 3 and Figure 3. I also show the estimated standard deviation for the measurement errors. All structural shocks are identified except for the markup shock in the non-tradable good. The posterior distributions for the measurement errors are mostly reasonably close to their priors, such that the measurement errors do not explain a sizeable share of the fluctuations observed in the data.\footnote{See Section 4.2 below.}

Figure 4 shows data for the variables used in the estimation (as in Figure 1) and the smoothed variables which represent the model fit. The model is able to reproduce the data very well, with the measurement errors absorbing some of the volatility, particularly for inflation, the import deflator and GDP growth. The estimates for credit data that are unobserved in the early 2000s show a strong rise in credit growth, consistent with the rise in house prices.
## Table 3: Prior and posterior distributions: Shock standard deviations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Prior</th>
<th>Posterior</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Std.dev.</td>
<td>Mean</td>
<td>10th</td>
</tr>
<tr>
<td>$\nu^H$</td>
<td>Inv. Gamma</td>
<td>0.100</td>
<td>0.100</td>
<td>0.0938</td>
<td>0.0728</td>
</tr>
<tr>
<td>$\nu^\beta$</td>
<td>Inv. Gamma</td>
<td>0.100</td>
<td>0.100</td>
<td>0.0642</td>
<td>0.0505</td>
</tr>
<tr>
<td>$\nu^m$</td>
<td>Inv. Gamma</td>
<td>0.010</td>
<td>0.050</td>
<td>0.0258</td>
<td>0.0213</td>
</tr>
<tr>
<td>$\nu^{ANT}$</td>
<td>Inv. Gamma</td>
<td>0.050</td>
<td>0.050</td>
<td>0.0252</td>
<td>0.0180</td>
</tr>
<tr>
<td>$\nu^{AH}$</td>
<td>Inv. Gamma</td>
<td>0.050</td>
<td>0.050</td>
<td>0.0993</td>
<td>0.0860</td>
</tr>
<tr>
<td>$\nu^{XD}$</td>
<td>Inv. Gamma</td>
<td>0.010</td>
<td>0.010</td>
<td>0.0624</td>
<td>0.0471</td>
</tr>
<tr>
<td>$\nu^{\mu NT}$</td>
<td>Inv. Gamma</td>
<td>0.010</td>
<td>0.010</td>
<td>0.0099</td>
<td>0.0046</td>
</tr>
<tr>
<td>$\nu^{\mu M}$</td>
<td>Inv. Gamma</td>
<td>0.010</td>
<td>0.010</td>
<td>0.0320</td>
<td>0.0192</td>
</tr>
<tr>
<td>$\nu^Y$</td>
<td>Inv. Gamma</td>
<td>0.0025</td>
<td>0.050</td>
<td>0.0050</td>
<td>0.0044</td>
</tr>
<tr>
<td>$\nu^{\Pi}$</td>
<td>Inv. Gamma</td>
<td>0.0010</td>
<td>0.050</td>
<td>0.0041</td>
<td>0.0036</td>
</tr>
<tr>
<td>$\nu^{R}$</td>
<td>Inv. Gamma</td>
<td>0.200</td>
<td>0.050</td>
<td>0.4918</td>
<td>0.4395</td>
</tr>
</tbody>
</table>

**Notes:** The last four columns on the right show the estimated mean, median and 10th and 90th percentiles of the posterior distributions.
Figure 4: Actual and smoothed (fitted) variables

Notes: All the data are de-meaned. The variables denoted as ‘Model’ are the smoothed estimates from the Kalman filter when parameters are at their posterior mode.
4 Properties of the estimated model

4.1 Impulse response functions (IRFs)

A monetary policy shock

Figure 5 shows the Bayesian dynamic responses to a euro area surprise monetary policy shock which raises the nominal rate. Since I use the Short Shadow Rate as the observed policy rate, this shock captures the effect of both conventional and unconventional monetary policy. Therefore, a typical monetary policy shock is estimated to reflect a stronger change in the nominal rate than that documented in the NAWM-II (Coenen et al., 2018). The effects of the shock on euro area variables are as expected, a decline in both output and prices which is persistent. The contraction in euro area output and inflation are similar to those in the NAWM-II, bottoming out at around 0.35% and 0.15 percentage points respectively.

The shock transmits to the Maltese economy through trade and financial channels. Demand for the export good falls leading to a drop in nominal wages in that sector, hitting households’ income. The drop in euro area inflation transmits via lower import prices, which lowers inflation in the price of the final consumption good. At the same time the rise in the nominal interest rate transmits to the local economy and, coupled with the decline in inflation, leads to a rise in (real) borrowing costs. The increase in the real deposit rate (not shown) is even larger, which stimulates a rise in deposits and lowers investment in capital and housing demand. Investment, house prices and household credit all fall, leading to a contraction in consumption and ultimately real output. The profile for consumption in Malta is very similar to the corresponding estimates for the euro area in Coenen et al. (2018), with a peak drop of just above -0.4% in the first year of the shock. The drop in domestic price pressures leads to a gradual decline in the real effective exchange rate (REER), which boosts exports after the initial decline. This ends up propping up real output in the medium term despite the monetary policy tightening in the euro area; a result consistent with VAR evidence documented in Gatt and Ruisi (2022), who find that a euro area monetary policy shock causes a drop in inflation but a rise in output.

A housing preference shock

Housing preference (demand) shocks play an important role in explaining fluctuations in house prices and consumption via collateral effects (Iacoviello, 2005). Figure 6 shows that the effect of a temporary but persistent rise in housing demand is a long-lasting rise in house prices. Credit rises and follows a hump-shaped response, driving an increase in consumption. This rise in consumption is attributable to impatient households, who are able to borrow more on the strength of rising house prices, despite a rise in real borrowing costs. The rise in consumption is close to the median SVAR response to an identified housing demand shock in Gatt and Ruisi (2020), although the rise in credit is about an order of magnitude higher here than in the

---

15 The interest rate pass-through is higher on impact than that estimated in Micallef et al. (2016), although their empirical estimates should be interpreted with caution as they represent unconditional policy rate changes, whereas here the shock is defined as an unexpected monetary tightening.

16 I verify that the collateral constraint remains binding throughout.
corresponding empirical response. Nevertheless, the peak consumption response is weaker than that estimated by Iacoviello and Neri (2010) for the US and by Nocera and Roma (2017) for Ireland, Italy and Spain. The shock creates upward pressure on domestic wages and prices, especially in the construction sector, which expands housing output in reaction to the higher demand for housing. It also stimulates a persistent increase in investment in capital used in
Figure 6: IRFs to a housing preference shock

Notes: The figure shows Bayesian dynamic responses of the variables to a structural shock, generated using 5,000 draws from the posterior distributions of the estimated parameters from each Metropolis-Hastings chain. The responses are in percentage deviations from the steady state level for all variables except for interest rates, which are in percentage point deviations. All inflation and interest rates are in annual terms.

construction in the medium term. These findings are consistent with those in Iacoviello and Neri (2010), who estimate a closed economy model of the US with collateralised borrowing by household and a construction sector that reacts to macroeconomic developments.

The rise in labour costs spills over to the production of the non-tradable and export goods, which leads to higher export prices and a drop in demand. Consequently, the net effect on real output is negative as the increase in housing construction is not enough to counter the drop in production in the other two sectors. This finding is also in line with the output response in Gatt and Ruisi (2020), who show that a housing demand shock leads to a small contraction in real output in the medium term. In that paper the authors conjecture that the drop in output
is likely driven by a deterioration in trade competitiveness. The estimated dynamic responses from MEDSEA-FIN confirm that this is a key transmission mechanism.

**Other demand shocks**

In Appendix C I show dynamic responses to other estimated shocks. An intertemporal preference (demand) shock raises consumption, but lowers investment and housing demand through substitution effects. The shock exerts upward pressure on price and wage inflation, which depresses competitiveness and results in a drop in exports. Given the strong increase in consumption, the net effect on real output is positive. A euro area demand shock transmits to the Maltese economy through higher export demand, stimulating the Maltese economy, despite also transmitting higher imported inflation. The ECB then raises interest rates to control the rise in euro area inflation, and this spills over to higher interest rates in the Maltese economy. Since the real interest rate falls, credit growth expands and fuels consumption in the medium term. The effects of the shock are long-lasting as the rise in euro area output is persistent, however price and wage pressures both domestically and in the euro area stabilise in the medium term.\(^{17}\)

**Supply shocks**

A productivity shock in the production of the non-tradable goods produces typical responses, it lowers prices and wages and increases investment, output, and consumption. The competitiveness gains lead to a rise in exports which boosts output further. House prices also rise although this is not accompanied by a rise in credit, as the increase in housing demand (and consumption) stems from savers. On the other hand, spurred by the rise in real borrowing costs and the appreciation in house prices, borrower (impatient) households liquidate their holdings of housing and use these resources to finance consumption. The stock of outstanding loans therefore falls.

Finally, an undesirable foreign supply shock raises euro area prices, and the ECB progressively raises interest rates to counter the drop in the real interest rate. The shock has a combined contractionary effect on euro area output. Nevertheless, Maltese exports increase following an improvement in the REER, given that the inflationary pressures do not transmit fully to the Maltese economy. This stimulates the domestic economy, with a rise in output that is broad-based and spread across consumption, investment and housing demand. Rising price and wage pressures then reduces the competitive position and dampen economic activity in the medium term, and the effects of the shock fade out. See Appendix C for more details.

**4.2 Historical decomposition**

Which shocks help explain the major fluctuations in house prices, credit and consumption, key variables that are tied together via the collateral constraint? The estimated model attributes a key role for the housing demand shock in driving fluctuations in house prices (74%) and credit (43%) in the long run (Table 4). This finding echoes the results of Liu et al. (2013) and Shirota

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\(^{17}\)Some of the persistence in domestic responses is due to endogenous persistence relating to the slow adjustment of the interest rate premium on foreign bonds. See Schmitt-Grohé and Uribe (2003) for more details.
(2018), who find that the housing demand shock explains about 90% and 75% of land price fluctuations, respectively, but is higher than the empirical estimates of Gatt and Ruisi (2020). Although the estimates in Liu et al. (2013) and Shirota (2018) strictly relate to land prices, these findings are comparable to those in MEDSEA-FIN given that a sizeable share of the house price in Malta reflects the value of land (and its scarcity). On the other hand, this contribution is much higher than that estimated by Iacoviello and Neri (2010). Nevertheless, the housing demand shock explains less than 1% of the movements in consumption in Malta. This result echoes the findings in Iacoviello and Neri (2010), who estimate a contribution of 0.3% for US consumption.

Table 4: Variance decomposition (% of long run fluctuations)

<table>
<thead>
<tr>
<th>H. Demand</th>
<th>Demand</th>
<th>Supply</th>
<th>Foreign</th>
<th>Financial</th>
<th>M.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>0.9</td>
<td>74.8</td>
<td>7.3</td>
<td>10.0</td>
<td>0.3</td>
</tr>
<tr>
<td>House prices</td>
<td>74.4</td>
<td>0.4</td>
<td>14.2</td>
<td>8.6</td>
<td>0.0</td>
</tr>
<tr>
<td>Credit</td>
<td>43.4</td>
<td>1.2</td>
<td>8.7</td>
<td>17.7</td>
<td>28.6</td>
</tr>
<tr>
<td>Inflation</td>
<td>1.5</td>
<td>1.2</td>
<td>37.2</td>
<td>56.9</td>
<td>0.3</td>
</tr>
<tr>
<td>GDP</td>
<td>0.4</td>
<td>1.4</td>
<td>44.3</td>
<td>48.7</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Notes: The table shows the asymptotic forecast error variance decomposition of the observed variables. The shocks are grouped as: H. Demand - housing preference shock, Demand - intertemporal preference shock, Supply - all technology and price mark-up shocks, Foreign - euro area demand, supply and monetary policy shocks, Financial - borrowing conditions shock, M.E. - measurement errors. Numbers may not add up to 100 due to rounding.

The role played by the housing demand shock varies somewhat from the VAR-based results in Gatt and Ruisi (2020), who show that this shock drives about 75% of house price fluctuations in the short term but drops to 50% at a 10-year horizon. Conversely, housing demand shocks explain almost none of the movements in consumption in the short term but account for about a third of movements at a 10-year horizon. The role that this shock plays in driving credit is about the same in both the DSGE-based (43%) and the VAR-based (50%) analysis. The differences could be due to the parsimonious (misspecified) structure of the VAR on one end, and the tight theoretical restrictions on the other. Nevertheless, I show below that these two methodologies appear to infer the same structural disturbance.

Demand shocks in the model drive primarily consumption, but have little effect on GDP given that they lead to opposite movement in exports due to competitiveness channels. Rather, almost half of movements in GDP are driven by supply shocks, with foreign shocks accounting roughly for the other half. The sizeable role of supply shocks in driving GDP in the long run is similar in magnitude to that estimated for the euro area in the NAWM-II. More than half of the movements in inflation are accounted for by foreign shocks, with supply shocks accounting for a little more than a third. The financial shock is, as expected, also important for driving credit, but has very little impact on the other observables.

Figure 7 decomposes the variations of house prices, credit and consumption over the past 20 years. The housing demand shock, as expected, was behind most of the ebb and flows of house prices around its long run average growth rate, and these episodes line up well with the implied
effects of the shock on credit and to a much lower extent, consumption. The financial shock seems to run counter to the housing demand shock through its dampening of credit in the mid-2000s and late 2010s and the upward pressure in the interim period. The shock could capture a countercyclical reaction of banks to the surge and drop in housing demand, by tightening and loosening credit standards at the bank level. Otherwise, it could also be an unidentified shock that is needed to account for the excessive variation of credit to the housing demand shock that is imposed by the structure of the model.

The historical effect of this shock on consumption has been low; positive during the mid-2000s, somewhat negative during the correction in the late 2000s (which coincides with the Great Recession) and then positive again in the mid-2010s. Intertemporal (time preference) shocks played a much more important role in driving consumption growth about its long run average. Foreign shocks played some role in specific times in driving these three variables. These are primarily monetary policy shocks, which reflect monetary tightening in the mid-2000s, considerable loosening via the transmission of conventional and unconventional policies following the financial and sovereign debt crises, and the slow unwinding of some of these policies more recently. This is at present one of only two studies that can shed some light on the transmission of monetary policy shocks to the Maltese economy; see Gatt and Ruisi (2022) for an empirical study.

The historical decomposition of other variables, shown in Appendix C, gives a more prominent role for supply shocks in driving the observed movements of inflation, dwelling investment and real GDP growth. In this regard, productivity shocks are the main contributors to the observed movements. The contribution of foreign shocks is also sizeable for GDP and prices, with euro area demand shocks playing an important role for GDP around the Great Recession and during the sovereign debt crisis. The period of low inflation in the mid-2010s in Malta is mainly attributed to negative euro area demand and lower imported inflation via ‘favourable’ supply shocks, as reflected in the import price deflator. The housing demand shock stands out as an important shock in the property market that transmits within the real estate sector to the rest of the economy, affecting the price of non-tradables and ultimately exerting upward price pressures (as shown in Section 4.1).
Figure 7: Historical drivers of macroeconomic fluctuations

Notes: The figure shows the historical decomposition of the observed variables into the structural shocks. All data is in year-on-year growth. The contributions are based on the posterior median of the structural parameters and shock variances. The shocks are grouped as: Housing Demand — housing preference shock, Demand — intertemporal preference shock, Supply — all technology and price mark-up shocks, Foreign — euro area demand, supply and monetary policy shocks, IV + ME — initial values and measurement errors.
4.3 Validation

I run two informal tests to judge the validity of the findings of the estimation presented in this paper. I first plot the (Kalman-smoothed) structural shocks from MEDSEA-FIN against comparable shocks inferred using SVARs. Here I draw from the results of Gatt and Ruisi (2022) to assess the euro area monetary policy and demand shocks, and the results of Gatt and Ruisi (2020) to compare the housing demand shock. Both of these SVARs are estimated on a low lag length and are identified using a mix of zero and sign restrictions on the impact response of the endogenous variables. By modern standards these are weak (set) restrictions imposed on a VAR with a low number of endogenous variables and a short lag structure, so the comparison with a highly stylised DSGE model with many cross-equation restrictions is not trivial (Canova and Ferroni, 2022).

Figure 8 plots the three normalised structural shocks at the mean values from their posterior distributions. The three shocks co-move significantly across the two methodologies, particularly the euro area demand shock and the domestic housing demand shock. This co-movement is remarkable in view of the differences between the underlying methodologies discussed above. The main differences for the monetary policy shock relate to the period just before and during the Great Recession. MEDSEA-FIN infers large unanticipated negative monetary policy shocks on the onset of the 2008 recession, whereas the SVAR infers a period of unanticipated monetary tightening. These differences can be reconciled by the fact that the SVAR contains the Composite Indicator of Systemic Stress (CISS) index (Hollo et al., 2012; Chavleishvili and Kremer, 2021), and is therefore more informed about the underlying state of the euro area economy. The large spike in the CISS index in 2009 purges the monetary policy response through the lens of the SVAR as a reaction rather than as a surprise. From 2011 on the two shocks series tell very similar stories.

A second test that can shed light on the external validity of the model is to assess the model-implied path of a macro variable that is not observed during the estimation. Given the strong link between the dynamics of exports and real output I show in the IRFs above, I plot the 90% Highest Posterior Density (HPD) of the Kalman-smoothed series for export growth from the model against the actual growth in exports. Figure 9 shows that although less noisy, the model-implied dynamics line up very well with the actual observed developments in exports. This confirms that MEDSEA-FIN can generate reasonable dynamics and therefore goes some way towards addressing the critique that DSGE models generally are only able to fit the data ‘in sample’.

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18 The ‘correlation’ of these shocks is understated in this graph, as the full shock distributions overlap much more than the mean.

19 I use a series for exports that excludes data for Special Purpose Entities.

20 See the discussion in Iacoviello (2015).
Figure 8: Structural shocks across macroeconomic frameworks

Notes: The structural shocks from MEDSEA-FIN are the smoothed shocks at the posterior mean of the parameters. The shocks from the Bayesian SVAR are the median from their underlying distribution. The SVAR that estimates the EA demand and monetary policy shocks is estimated from 2004Q2 to 2019Q4, while the SVAR that estimates the housing demand shock is estimated from 2000Q3 to 2019Q4. All shocks are normalized.
Figure 9: Actual and model-implied time series path for exports

Notes: The actual data for exports exclude Special Purpose Entities and the growth rate is demeaned. The model-implied series is the Kalman-smoothed path covered by the 90% Highest Posterior Density.
5 The effect of the pandemic on the estimated parameters

All the results shown so far are based on data estimated up to 2019Q4, and therefore exclude the more recent data that cover the COVID-19 pandemic. However, the economic fluctuations observed over the pandemic raise concerns about the usefulness of including data for 2020 and 2021 in the information set. To control for these huge shocks, Cardani et al. (2022) and Lenza and Primiceri (2022) propose the use of a ‘heteroscedastic filter’ which deterministically imposes a time-varying variance on a subset of the structural shocks. Cardani et al. (2022) show that their application on the EU Commission’s Global Multi-country model is able to capture the huge ‘COVID shocks’ as forced saving and labour hoarding shocks, while keeping all other inference on the past unchanged.

Despite this success, it remains unclear whether this approach is in itself sufficient to provide a full narrative of the effects of the pandemic, especially since some scarring effects may be long-lasting and delayed, and therefore still to be observed. Exploring further modifications to the model (or waiting for more data) is beyond the scope of this paper. However, to justify such future efforts I show below the implications of ignoring the unprecedented swings in the data observed during the pandemic and I re-estimate the model using data up to 2021Q4. I show the resulting posterior distributions in Figures 10 and 11 and compare them with those from the benchmark estimates, and I also show the implied historical decomposition for house prices, credit and consumption in Figure 12 below.

Although the posterior distribution of several parameters are not affected, a few are, and significantly. In particular, the distribution for the persistence parameter on the intertemporal preference shock shifted to the left, and the corresponding shock standard deviation to the right. This is driven by the need for a shock that can explain the sudden and large drop in consumption (and GDP) but which so far has had little to no effect on house prices and credit.21 Similarly, the persistence of the euro area demand shock falls and its variance rises, as it is the key shock that explains the observed movements in euro area GDP and prices (not shown). Euro area supply shocks also play a role, especially towards the second half of 2021 when inflation starts rising. However, in this case both the shock persistence and its variance are higher, reflecting the more progressive nature of inflationary pressures as supply bottlenecks hit.

This exercise illustrates the sensitivity of the estimation results to an additional 8 observations when they involve severe macroeconomic gyrations. Although the sample used in the benchmark estimation is admittedly relatively short, Cardani et al. (2022) show that working with a slightly longer sample does not improve the outcome, as they report a noticeable drop in estimated consumption habits and the persistence of the intertemporal preference shock. On the other hand, restricting the information set to 2019Q4 and decomposing the period thereafter does not yield satisfactory historical decompositions due to weaker inference (not shown). Modifications along those proposed in the literature and perhaps more are likely required before the pandemic data can be included in the estimation information set.

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21 In fact, raw credit growth expanded further since the onset of the pandemic, from an average yearly growth rate of 3.8% (2016–2019) to 5.8% in the period 2020–2021. This was likely partly due to government stimulus in the form of reduced stamp duty on the purchase of property, in an effort to keep this sector of the economy, and all other ancillary sectors, from suffering a collapse due to the fall on aggregate economic activity.
Notes: The figure shows the prior and posterior distributions as fitted kernel densities from two Metropolis-Hastings chains of length 5 million draws each. ‘Benchmark’ is the model estimated in Section 3, ‘COVID’ denotes the benchmark model estimated using data up to 2021Q4.
Figure 11: Prior and posterior distributions: Shock and measurement error standard deviations

Notes: The figure shows the prior and posterior distributions as fitted kernel densities from two Metropolis-Hastings chains of length 5 million draws each. ‘Benchmark’ is the model estimated in Section 3, ‘COVID’ denotes the benchmark model estimated using data up to 2021Q4.
Figure 12: Historical drivers of macroeconomic fluctuations extended over the pandemic period
Notes: The figure shows the historical decomposition of the observed variables into the structural shocks. All data is in year-on-year growth. The contributions are based on the posterior median of the structural parameters and shock variances. Shock groups: Housing Demand – housing preference shock, Demand - intertemporal preference shock, Supply - all technology and price mark-up shocks, Foreign - euro area demand, supply and monetary policy shocks, IV + ME – initial values and measurement errors.
6 Conclusion

This paper documents the first attempt at estimating a medium-scale DSGE model using Maltese data. It uses MEDSEA-FIN, a DSGE model with household heterogeneity, housing and banks, and estimates a subset of the parameters that control the dynamic responses to structural shocks and their variances. It finds that the data is informative on several parameters, and the implied dynamic responses to shocks are reasonable and match findings from other studies. The housing demand shock and foreign shocks were behind the key cycles observed in the data, and they provide a reasonable historical narrative. Besides these results, the contribution of this paper is to yield an estimated DSGE model that can be used for policy analysis on matters related to housing and macroprudential policy.

Nevertheless, there are several avenues for future research and model development. First, the model requires more development in order to accommodate future estimation attempts using data that span the pandemic period and beyond. In this regard, the approach and resulting algorithm developed in Cardani et al. (2022) is a promising way forward. Second, although I develop the foreign block from exogenous AR(1) processes into a self-contained 3-equation equilibrium system, improvements in its structure can be made along the Smets and Wouters (2003) model. Third, the bank block can be developed further along the lines of Gerali et al. (2010), such that domestic banks are monopolistically competitive and the interest rate response is more sluggish. Related to this, more comprehensive modelling of bank capital ratios which correspond to policy tools would make the model more amenable to macroprudential policy analysis. These strands of work are left for future research.
References


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Appendix A  Model equations

This appendix lists all the equations of the model. Refer to Gatt et al. (2020) for more details.

Households – savers

\[ K_{s,j,t}^{NT} = (1 - \delta_{KNT})K_{s,j,t-1}^{NT} + I_{s,j,t}^{NT} \left[ 1 - \frac{\xi_{s,j,t}}{2} \left( \frac{I_{s,j,t}^{NT}}{I_{s,j,t-1}^{NT}} - 1 \right) \right] \]  \hspace{1cm} (A.1)

\[ K_{s,j,t}^{H} = (1 - \delta_{KH})K_{s,j,t-1}^{H} + I_{s,j,t}^{H} \left[ 1 - \frac{\xi_{s,j,t}}{2} \left( \frac{I_{s,j,t}^{H}}{I_{s,j,t-1}^{H}} - 1 \right) \right] \]  \hspace{1cm} (A.2)

\[ \lambda_{s,t} = \varepsilon_{t}^H \left( \frac{C_{s,t} - \Gamma_{s,t-1}}{(1 - \chi)} \right)^{\sigma} \]  \hspace{1cm} (A.3)

\[ \lambda_{s,t} = \beta_{s} E_t \left\{ R_{t} \lambda_{s,t+1} \frac{\Pi_{H_{s,t}}^{C}}{\Pi_{H_{s,t}}^{C}} \right\} \]  \hspace{1cm} (A.4)

\[ \lambda_{s,t} = \beta_{s} E_t \left\{ \frac{R_{t} \lambda_{s,t+1}^{H}}{\Pi_{H_{s,t}}^{C}} \right\} \]  \hspace{1cm} (A.5)

\[ p_{t}^{KNT} = p_{t}^{KN} \left[ 1 - \frac{\xi_{t}^{N}}{2} \left( \frac{I_{s,t}^{N}}{I_{s,t-1}^{N}} - 1 \right) \right] - r_{t+1}^{KNT} \]  \hspace{1cm} (A.6)

\[ p_{t}^{KH} = \beta_{s} E_t \left\{ \frac{\lambda_{s,t+1}^{KH}}{\lambda_{s,t}} \left( 1 - \delta_{KNT}p_{t+1}^{KNT} + r_{t+1}^{KNT} \right) \right\} \]  \hspace{1cm} (A.7)

\[ p_{t}^{KH} = \beta_{s} E_t \left\{ \frac{\lambda_{s,t+1}^{KH}}{\lambda_{s,t}} \left( 1 - \delta_{KH}p_{t+1}^{KH} + r_{t+1}^{KH} \right) \right\} \]  \hspace{1cm} (A.8)
Households – borrowers

\[ C_{b,j,t} + p_t^H \left( H_{b,j,t} - (1 - \delta_H)H_{b,j,t-1} \right) + \frac{R_{t-1}^L L_{b,j,t-1}}{\Pi^L_t} \]

\[ = \sum_m w_{b,j,t}^m N_{b,j,t} \left( 1 - AC_{b,j,t}^W \right) + L_{b,j,t} - \frac{T_{b,j,t}}{R^C_t} + \frac{\pi_{B,t}}{1 - \omega} \quad \text{(A.11)} \]

\[ L_{b,t} = \rho_L \frac{L_{b,t-1}}{\Pi^C_t} + (1 - \rho_L) m_{e_t}^m \mathbb{E}_t \left\{ \frac{p_{t+1}^H H_{b,t} \Pi_{t+1}^C}{R^L_{t+1}} \right\} \quad \text{(A.12)} \]

\[ \lambda_{b,t} = \varepsilon_t^B \left( \frac{(1 - \chi)}{(C_{b,t} - \Gamma_{b,t-1})} \right) \sigma \quad \text{(A.13)} \]

\[ \lambda_{b,t} = \beta_t \mathbb{E}_t \left\{ \frac{R^L_t \lambda_{b,t+1} - \rho_L \lambda_{t+1}^L}{\Pi_{t+1}^C} \right\} + \lambda^L_t \quad \text{(A.14)} \]

\[ \varepsilon_t^B \frac{p_t^H}{H_{b,t}} = p_t^H \lambda_{b,t} - \beta_t (1 - \delta_H) \mathbb{E}_t \left\{ \frac{p_{t+1}^H \lambda_{b,t+1}}{R^L_{t+1}} \right\} \]

\[ - (1 - \rho_L) \lambda_{b,t} m_{e_t}^L \mathbb{E}_t \left\{ \frac{p_{t+1}^H \Pi_{t+1}^C}{R^L_{t+1}} \right\} \quad \text{(A.15)} \]

The labour market – packers

\[ w_{i,t}^m \left( \mu^W - 1 \right) + \xi^m W \Phi_{i,t}^m \left( \Phi_{i,t}^m - 1 \right) - \xi^m W \beta_t \mathbb{E}_t \frac{\lambda_{i,t+1}^m}{\lambda_{i,t}} N_{i,t+1}^m \frac{\Pi_{i,t+1}^W \Phi_{i,t+1}^m (\Phi_{i,t+1}^m - 1)}{N_{i,t}^m} \]

\[ = \frac{\mu^W}{\varepsilon_t^L} \varepsilon_t^L \left( N_{i,t}^m \right)^{1 - \mu^W} \left[ \left( N_{i,t}^W \right)^{1 + \xi} + \left( N_{i,t}^H \right)^{1 + \xi} + \left( N_{i,t}^W \right)^{1 + \xi} \right] \quad \text{(A.16)} \]

\[ N_{s,t}^m = \varpi \left( \frac{w_{s,t}^m}{w_{i,t}^m} \right)^{-\mu^W} N_{i,t}^m \quad \text{(A.17)} \]

\[ N_{b,t}^m = (1 - \varpi) \left( \frac{w_{b,t}^m}{w_{i,t}^m} \right)^{-\mu^W} N_{i,t}^m \quad \text{(A.18)} \]

\[ w_{i,t}^m = \left( \varpi \frac{w_{s,t}^m}{w_{b,t}^m} \right)^{1 - \mu^W} + (1 - \varpi) \left( \frac{w_{b,t}^m}{w_{s,t}^m} \right)^{1 - \mu^W} \quad \text{(A.19)} \]

for \( i \in \{s, b\} \) in sector \( m \in \{NT, XD, H\} \), where \( \Phi_{i,t}^m = \Pi_t^{W,m} / \left( \left( \Pi_{i,t-1}^{W,m} \right)^{1 - \mu^W} \Pi_t^{1 - \mu^W} \right) \).
The real estate sector

\[
Y_t^H = \left( \wp_{H}^{M} (N_t^H)^{\frac{\mu_H-1}{\mu_H}} + (1 - \alpha_H)^{\frac{\mu_H-1}{\mu_H}} (A_t^H K_{t-1}^H)^{\frac{\mu_H-1}{\mu_H}} \right)^{\frac{\mu_H}{\mu_H-1}} \tag{A.20}
\]

\[
N_t^H = \alpha_H \left( \frac{P_t^H}{W_t^H} \right)^{\frac{\mu_H}{\mu_H-1}} Y_t^H \tag{A.21}
\]

\[
K_{t-1}^H = (1 - \alpha_H) \left( \frac{P_t^H}{R_t^H} \right)^{\frac{\mu_H}{\mu_H-1}} A_t^H (\mu_H-1) Y_t^H \tag{A.22}
\]

\[
P_t^H = \left( \alpha_H (W_t^H)^{1-\mu_H} + (1 - \alpha_H) (R_t^H)^{1-\mu_H} A_t^H (\mu_H-1)^2 \right)^{\frac{1}{\mu_H}} \tag{A.23}
\]

\[
\tilde{H}_t = (1 - \delta_H) \tilde{H}_{t-1} + Y_t^H \tag{A.24}
\]

Banks

\[
D_t + \frac{R_{t-1}L_{t-1}}{\Pi_t^C} = L_t + \frac{R_{t-1}D_{t-1}}{\Pi_t^C} + \pi_{B,t} - \frac{\xi_B}{2} (L_t - L_{t-1})^2 - \frac{\xi_B}{2} (D_t - D_{t-1})^2 \tag{A.25}
\]

\[
D_t = (1 - c_{B,t}) L_t \tag{A.26}
\]

\[
\beta_B R_t^B + (1 - c_{B,t}) \lambda_t^B + \beta_B \xi_t \left\{ \xi_B \left( \frac{L_t - L_{t-1}}{L_{t-1}} \right) \right\} = 1 + \frac{\xi_B}{L} (L_t - L_{t-1}) \tag{A.27}
\]

\[
1 + \beta_B \xi_t \left\{ \xi_B \left( \frac{D_t - D_{t-1}}{D_{t-1}} \right) \right\} = \beta_B R_t^B + \lambda_t^B + \frac{\xi_B}{D} (D_t - D_{t-1}) \tag{A.28}
\]

Manufacturers – producers of NT

\[
Y_t^{NT} = A_t^{NT} (K_{t-1}^{NT})^{1-\gamma_{NT}} (N_t^{NT})^{\gamma_{NT}} \tag{A.29}
\]

\[
W_t^{NT} N_t^{NT} = \gamma_{NT} M C_t^{NT} Y_t^{NT} \tag{A.30}
\]

\[
R_t^{KNT} K_{t-1}^{NT} = (1 - \gamma_{NT}) M C_t^{NT} Y_t^{NT} \tag{A.31}
\]

\[
P_t^{NT} \left( \mu_t^{NT} - 1 \right) + \xi_t^{NT} (\Phi_t^{NT} - 1) \Phi_t^{NT} - \xi_t^{NT} \xi_t L_t \left( \frac{Y_{t+1}^{NT} (\Phi_{t+1}^{NT} - 1) \Phi_{t+1}^{NT}}{Y_t^{NT}} \right) = \mu_t^{NT} M C_t^{NT} \tag{A.32}
\]

where \( \Phi_t^{NT} = \Pi_t^{PNT} / \left( (\Pi_{t-1}^{PNT})^{1-t_{\Phi}^{NT}} \right) \).

Manufacturers – importers

\[
M C_{t,t}^{M} = P_t^{M} S_t \tag{A.33}
\]

\[
P_t^{M} \left( \mu_t^{M} - 1 \right) + \xi_t^{M} (\Phi_t^{M} - 1) \Phi_t^{M} - \xi_t^{M} \xi_t L_t \left( \frac{Y_{t+1}^{M} (\Phi_{t+1}^{M} - 1) \Phi_{t+1}^{M}}{Y_t^{M}} \right) = \mu_t^{M} M C_t^{M} \tag{A.34}
\]

where \( \Phi_t^{M} = \Pi_t^{PMT} / \left( (\Pi_{t-1}^{PMT})^{1-t_{\Phi}^{MT}} \right) \).
Manufacturers – producers of XD

\[ Y_t^{XD} = A_t^{XD} (K_{t-1}^{XD})^{1-\gamma_{XD}} (N_t^{XD})^{\gamma_{XD}} \]  \hspace{1cm} (A.35)

\[ W_t^{XD} N_t^{XD} = \gamma_{XD} MC_t^{XD} Y_t^{XD} \]  \hspace{1cm} (A.36)

Final sellers – local market

\[ Y_{t,NT}^z = (1 - \alpha_z) \left( \frac{P_{NT}^z}{P_t^z} \right)^{-\eta_z} Y_t^z \]  \hspace{1cm} (A.37)

\[ Y_{t,M}^z = \alpha_z \left( \frac{P_{M}^z}{P_t^z} \right)^{-\eta_z} Y_t^z \]  \hspace{1cm} (A.38)

\[ P_t^z = \left( (1 - \alpha_z) \left( \frac{P_{NT}^z}{P_t^z} \right)^{1-\eta_z} + \alpha_z \left( \frac{P_{M}^z}{P_t^z} \right)^{1-\eta_z} \right)^{-\frac{1}{\eta_z}} \]  \hspace{1cm} (A.39)

for \( z \in \{C, I\} \).

Final sellers – export market

\[ Y_t^{XD} = (1 - \alpha_X) Y_t^X \]  \hspace{1cm} (A.40)

\[ Y_t^{MX} = \alpha_X Y_t^X \]  \hspace{1cm} (A.41)

\[ MC_t^X = (1 - \alpha_X) MC_t^{XD} + \alpha_X P_t^M \]  \hspace{1cm} (A.42)

\[ P_t^{XW} \left( \left( \mu_t^{XD} - 1 \right) + \xi_t^{XW} (\Phi_t^{XW} - 1) \Phi_t^{XW} - \xi_t^{XW} E_t A_{t,t+1} \frac{Y_{t+1}^X}{Y_t^X} (\Phi_{t+1}^{XW} - 1) \Phi_{t+1}^{XW} \right) = \mu_t^{XD} MC_t^X \]  \hspace{1cm} (A.43)

\[ P_t^X = P_t^{XW} + \theta P_{NT}^X \]  \hspace{1cm} (A.44)

where \( \Phi_t^{XW} = \Pi_t^{XW} / \left( \left( \Pi_{t-1}^{XW} \right) \Phi_{t-1}^{XW} \left( \Pi_{t-1}^{XW} \right)^{1-\xi_{t-1}^{XW}} \right) \).

\[ Y_t^X = \left( \frac{P_t^X}{\sum_{t=1}^T P_t^X} \right)^{-\eta_X} Y_t^* \]  \hspace{1cm} (A.45)

Policy authorities

\[ m_t = m_{t-1}^m \left( \frac{P_t^C L_t}{Y_t^{t-1}} \right)^{-\tau_m (1-\rho_m)} \]  \hspace{1cm} (A.46)

\[ c_{B,t} = c_{B,t} \left( \frac{P_t^C L_t}{Y_t^{t-1}} \right)^{-\tau_B (1-\rho_B)} \]  \hspace{1cm} (A.47)

\[ T_t = P_{NT}^N Y_t^G \]  \hspace{1cm} (A.48)

\[ \frac{P_{NT}}{Y_t^G} = g_t Y \]  \hspace{1cm} (A.49)
Trade

\[ R_t = R_t^* \frac{E_t S_{t+1}}{S_t} e^{\phi_t} \]  
\[ \phi_t = \rho_\phi \left( \frac{P_t^* S_t B_t^*}{4Y_t} - \xi_t \right) + \varepsilon_t^{\phi} \]  
\[ B_t^* = \frac{B_{t-1}^* R_{t-1}^*}{\Pi_t^*} - TB_t \]  
\[ TB_t = P_t^X Y_t^X - P_t^M Y_t^M \]  
\[ TOT = \frac{P_t^X}{P_t^M} \]  

The euro area block

\[ \tilde{Y}_t^* = \phi_1 Y_t^* \tilde{Y}_t^* + \left(1 - \phi_1 Y_t^* \right) \tilde{Y}_{t-1}^* - \phi_2 Y_t^* \left( R_t^* - R^* - E_t \Pi_{t+1}^* - \Pi^* \right) + \varepsilon_t^{Y_t^*} \]  
\[ \log(P_t^*) = \rho_{P^*} \log(P_{t-1}^*) + \phi_{P^*} \log(\tilde{Y}_t^*) + \varepsilon_t^{P_t^*} \]  
\[ R_t^{a,*} = \rho_{R^*} R_{t-1}^{a,*} + (1 - \rho_{R^*}) \left( R_t^{a,*} + (1 - \tau) R_t^{a,*} \right) + \nu_t^{R_t^{a,*}} \]  

Aggregation and market clearing

\[ P_t^Y Y_t = P_t^C C_t + P_t^I I_t + P_t^{NT} Y_t^G + P_t^X Y_t^X - P_t^M Y_t^M \]  
\[ Y_t = P_t^C C_t + P_t^I I_t + P_t^{NT} Y_t^G + P_t^X Y_t^X - P_t^M Y_t^M \]  
\[ Y_t^C = C_t \]  
\[ Y_t^I = I_t \]  
\[ Y_t^{NT} = Y_t^{C,NT} + Y_t^{I,NT} + Y_t^G \]  
\[ Y_t^M = Y_t^{C,M} + Y_t^{I,M} + Y_t^{MX} \]  
\[ \tilde{H}_t = \varpi H_{s,t} + (1 - \varpi) H_{b,t} \]  
\[ C_t = \varpi C_{s,t} + (1 - \varpi) C_{b,t} \]  
\[ D_t = \varpi D_{s,t} \]  
\[ L_t = (1 - \varpi)L_{b,t} \]  
\[ B_t^* = \varpi B_{s,t}^* \]  
\[ I_t = \varpi (I_{s,t}^N + I_{s,t}^H) + \delta K_t^{XD} \]  
\[ K_t^{NT} = \varpi K_{s,t}^{NT} \]  
\[ K_t^H = \varpi K_{s,t}^H \]  
\[ T_t = \varpi T_{s,t} + (1 - \varpi) T_{b,t} \]  
\[ T_{b,t} = \nu_T T_t \]
Shock processes

Preference shocks (3)

\[ \log(\varepsilon_i^t) = (1 - \rho_{\varepsilon}) \log(\varepsilon_i^{t-1}) + \rho_{\varepsilon} \varepsilon_i^t, \quad i \in \{\beta, H, N\} \] (A.74)

Technology shocks (3)

\[ \log(A_{\ell}^t) = \rho_{A_{\ell}} \log(A_{\ell}^{t-1}) + (1 - \rho_{A_{\ell}}) \log(A_{\ell}^t) + \nu_{\ell}^t, \quad \ell \in \{A^H, A^{NT}, A^{XD}\} \] (A.75)

Policy shocks (1)

\[ \log(g_t) = (1 - \rho_g) \log(g_{t-1}) + \rho_g g_t + \nu_g^t \] (A.76)

Mark-up shocks (3)

\[ \log(\mu_m^t) = \rho_{\mu,m} \log(\mu_m^{t-1}) + (1 - \rho_{\mu,m}) \log(\mu_m^t) - \nu_{\mu,m}^t, \quad m \in \{NT, M, XD\} \] (A.77)

Foreign shocks (5)

\[ \varepsilon_n^n = \rho_{\varepsilon^n} \varepsilon_{n-1}^n + \nu_n^n, \quad n \in \{Y^*, P^*\} \] (A.78)

\[ \nu_{\varepsilon^n}^{Y^*, P^*} \sim i.i.d \] (A.79)

\[ \log(K_{XD}^t) = (1 - \rho_{K XD}) \log(K_{XD}^{t-1}) + \rho_{K XD} \log(K_{XD}^t) + \nu_{K XD}^t \] (A.80)

\[ \varepsilon_{\phi}^t = \rho_{\varepsilon_{\phi}} \varepsilon_{\phi-1}^t + \nu_{\phi}^t \] (A.81)

Appendix B  Data

B.1 Population

Population data are available in annual frequency from Eurostat. I interpolate the data within the year using a cubic spline, such that the end-of-year value is consistent with the official number.

B.2 National accounts components

Real consumption, investment in dwellings, and GDP are defined as the ESA 2010 chain-linked volumes. The vintage relates to NSO news release 037/2021 which reports data up to 2021Q4 and therefore the data used in all estimations are subject to revisions. I express these variables in per capita terms based on the quarterly population series I construct. The import deflator corresponds to the ratio of total nominal and real imports.
B.3 Prices of final consumption goods and non-tradables

I use the HICP index for the price of the final consumption good, and the HICP services index to proxy the price of the non-tradable goods. Both are sourced from Eurostat.

B.4 House prices

House prices are measured using the Central Bank of Malta’s house price index, which is based on advertised property prices. I re-base the index to 100 in 2000 and deflate it using the HICP excluding energy index, obtained from Eurostat.

B.5 Total household credit

Total credit to households for the period 2003Q4–2019Q4 is obtained from the Central Bank of Malta (OMFI loans to residents of Malta by economic activity) using end-of-period values. I fill in missing values for data prior to this period by obtaining corresponding series from the STREAM database, which is the Central Bank of Malta’s macroeconometric model, and splicing the missing values for OMFI loans back to 2000Q1 based on quarterly growth rates. I then divide total credit by the population series.

B.6 Euro area GDP and inflation

Euro area GDP (19 countries) is seasonally and calendar day adjusted, expressed in chain-linked 2015 volumes. Euro area inflation is based on the headline HICP. Both were obtained from Eurostat.

B.7 ECB policy rate

I use the Short Shadow Rate (Krippner, 2013, 2020) to proxy both conventional and unconventional ECB monetary policy. I obtain the series from Leo Krippner’s website https://www.ljkmfa.com/.
Appendix C  Other figures

Figure C.1: IRFs to an intertemporal preference shock

Notes: The figure shows Bayesian dynamic responses of the variables to a discount factor shock, generated using 5,000 draws from the posterior distributions of the estimated parameters from each Metropolis-Hastings chain. The responses are in percentage deviations from the steady state level for all variables except for interest rates, which are in percentage point deviations. All inflation and interest rates are in annual terms.
Figure C.2: IRFs to a EA demand shock

Notes: The figure shows Bayesian dynamic responses of the variables to euro area demand shock, generated using 5,000 draws from the posterior distributions of the estimated parameters from each Metropolis-Hastings chain. The responses are in percentage deviations from the steady state level for all variables except for interest rates, which are in percentage point deviations. All inflation and interest rates are in annual terms.
Figure C.3: IRFs to a productivity shock in the non-tradable sector

Notes: The figure shows Bayesian dynamic responses of the variables to euro area demand shock, generated using 5,000 draws from the posterior distributions of the estimated parameters from each Metropolis-Hastings chain. The responses are in percentage deviations from the steady state level for all variables except for interest rates, which are in percentage point deviations. All inflation and interest rates are in annual terms.
Figure C.4: IRFs to a euro area supply shock

Notes: The figure shows Bayesian dynamic responses of the variables to euro area demand shock, generated using 5,000 draws from the posterior distributions of the estimated parameters from each Metropolis-Hastings chain. The responses are in percentage deviations from the steady state level for all variables except for interest rates, which are in percentage point deviations. All inflation and interest rates are in annual terms.
Figure C.5: Historical drivers of macroeconomic fluctuations

Notes: The figure shows the historical decomposition of the observed variables into the structural shocks. All data is in year-on-year growth. The contributions are based on the posterior median of the structural parameters and shock variances. The shocks are grouped as: Housing Demand – housing preference shock, Demand - intertemporal preference shock, Supply - all technology and price mark-up shocks, Foreign - euro area demand, supply and monetary policy shocks, IV + ME – initial values and measurement errors.
Figure C.6: Historical drivers of macroeconomic fluctuations extended over the pandemic period. The figure shows the historical decomposition of the observed variables into the structural shocks. All data is in year-on-year growth. The contributions are based on the posterior median of the structural parameters and shock variances. The shocks are grouped as: Housing Demand – housing preference shock, Demand - intertemporal preference shock, Supply - all technology and price mark-up shocks, Foreign - euro area demand, supply and monetary policy shocks, IV + ME – initial values and measurement errors.