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# Investigating potential output using the Hodrick-Prescott filter: An Application for Malta[1]

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# Abstract

Modern economics assumes that in the long run an economy develops in a balanced way, with full employment of resources and a constant inflation rate. The output level thereby achieved is called 'potential output'. Knowing the extent of the output gap, or the deviation from this equilibrium, is crucial both to guide discretionary policy and to help set an environment conducive to higher long-term growth. Consequently substantial research has been carried out to devise methods to determine the equilibrium output of an economy.

This paper, after reviewing the most used method from the two main branches of empirical investigation, namely the Hodrick-Prescott (HP) filter and the production function, argues that they both suffer from significant failings when applied to very small and open economies like Malta. The latter tend to have series that exhibit a number of pronounced trends, large fluctuations and recurrent structural breaks. Lack of data seriously constrains the production function approach in such a setting. However, non-structural methods like the HP filter tend to exhibit very pronounced changes in the output gap that are out of line with the theoretical idea of equilibrium, with results also affected significantly by shocks to data. In turn, these reflect the small size of the economy rather than actual changes in potential output.

Two suggestions are put forward. The first one involves an innovative application of the standard filter, whereby the upper and lower bounds of a series are defined and equilibrium is determined as a weighted average of the HP filter applied separately on these bounds. This can result in a smoother output gap series with the possibility of long-term deviations from equilibrium. The second suggestion involves an integration of structural features into the standard HP filter. The modified or generalised HP filter would allow researchers to set limits on the impact exerted by structural or temporary shocks and to allow for the possibility of having lengthy periods of disequilibria.

JEL Classification: B41, C18, E32, F41.

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# Introduction

Modern economics posits that in the long run an economy develops in a balanced way, with full employment of resources and a constant inflation rate. The output level thereby achieved is called ‘potential output’ as it is the highest output level achievable without introducing disequilibria in the production factor markets. The business cycle can be described in relation to this potential output, with the percentage difference between the observed output and the potential one being known as the ‘output gap’.

Potential output is unobservable and hence needs to be estimated. A number of methods have been developed in this regard, with two main branches emerging in the literature. On the one hand, statistical de-trending methods attempt to extract potential output using past values of the real Gross Domestic Product (GDP) series. The line of reasoning underlying these methods is essentially neo-classical as the economy’s dynamics are assumed to gravitate towards equilibrium, so that past values of actual GDP give a good indication of potential output. By contrast, the other category of models is based on the Keynesian precept that the economy can deviate from equilibrium for long periods of time. As a result, past observations of real GDP provide little indication of the extent of the output gap, and a structural model, most frequently a production function, must be developed to estimate the potential capacity of the economy.

These two empirical schools, described in the first section of this paper, differ fundamentally. The structural methods, if they are not correctly specified or are based on an inaccurate assessment of factors of production, can provide a rather incorrect view of potential output. Given the difficulty of estimating the volume of factors like capital, the value of human capital and the natural rate of unemployment, researchers can arrive at very different measures of the output gap for the same country. In view of the great degree of subjectivity that accompanies such structural assessments of potential GDP, many institutions opt to use the filtering methods in their work. These methods have the added benefit that they can be applied uniformly in cross-country studies. Nevertheless the use of these filtering methods remains controversial as they tend to be too inflexible and at times they fail to adjust for special occurrences. Thus, for example, when the GDP series in many countries (especially those dependent on tourism) dropped substantially in 2001 because of the September 11 attacks, these methods tended to show a drop in potential output that was quickly reversed in the following years. Though for large countries these kinds of shocks tend to be rare occurrences, the same cannot be said for small open economies. Hence the use of standard filtering methods might not be that appropriate in such cases. Yet, the data sources in these economies tend to be too limited to be able to carry out detailed structural assessments of potential output. In this light, for such countries it would be optimal to use filtering methods that are adjusted to take into account their special characteristics.

This paper will attempt to tackle this issue by looking at the experience of the Maltese economy. Malta has been a success story in terms of economic growth. In the period 1960-90 Malta was among only six countries (the others were the so-called Asian tigers and Botswana) to achieve an annual average GDP growth rate higher than 5% every year.[2] More recently, in the 2000s the growth rate slowed to an annual average of 1.8%. Although the Maltese economy is well diversified, some sectors are very dependent on foreign trade, with exports amounting to 90% of real GDP. Moreover given its small size,[3] its real output series is prone to frequent spikes and large drops due to one-off factors, like the fulfilment of a major export order or the purchase of large capital equipment. Therefore Malta presents an excellent example of a country where the application of standard filters could provide policymakers with an incorrect value for potential output. In the second section of this paper, a modification of the HP filter is proposed to enable it to take better into account underlying characteristics of a series. In essence, this method tries to make estimates of potential output less sensitive to one-off changes in GDP and also allows for long-term deviations from potential.

# 1.0 Potential output – Theory and empirical application

This section first outlines the theories underlying the concept of potential output and then proceeds to describe how economists have tried to apply these theoretical frameworks in practice. The differences between the main schools of thought that have formed on this subject reflect fundamental divergences in opinion about long-term macroeconomic behaviour. While there are common features between studies based on the various methods, any survey of literature reveals that there are fundamental divergences between results presented for the same economies. Chagny & Dopke (2001), for instance, report that ‘the correlation of output gaps calculated with different methods is generally low, the methods imply different turning points, and the estimated level of the output gap differs greatly’.

## *1.1 Potential output – The concept and its relevance*

In his seminal paper, Okun (1962) defined potential GDP as the answer to the question: “How much output can the economy produce under conditions of full employment?” This definition is important in that it clarifies an important issue, namely that potential output is not the level of output that is attainable, in a mechanical sense, with the maximum utilisation of the factors of production.[4] The full

[2] Alesina & Spolaore (2003). The Penn World Table Version 2012 also suggests this average growth rate.
[3] Malta’s real GDP in 2011 stood at EUR5.6 billion, or 0.07% of the euro area’s GDP.
[4] See European Central Bank (2000).

use of all the factors of production is not economically feasible, as marginal costs rise steeply (and exceed marginal revenues) at high levels of factor utilisation. Therefore potential output implies the optimal use of resources in the light of prevailing economic constraints in factor markets, such as the prevailing natural rate of unemployment. In effect, as De Masi (1997) states, potential GDP is 'the maximum output an economy can sustain without generating a rise in inflation'.

Inherent in this definition is the idea that any measure of potential output has a short-term perspective, in that it takes facts about the economy as they exist. Thus technological knowledge, the capital stock, natural resources, the skill and size of the labour force are treated as data, rather than as variables. Defined in this way, the output gap is an aggregate measure of resource strain in the economy, and can serve as a device for policymakers to represent how "hot" or "cold" an economy is at any particular time, and to forecast inflationary pressures.[5] It is also an indicator of the volatility of economic activity, showing whether resources are alternating between periods of substantial use to others of significant slack, or whether the economy is moving smoothly.[6]

This idea of potential output has been interpreted analytically in two ways by macroeconomists. One school, which harks back to the classical tradition, views the business cycle as fluctuations around a long-term trend. The other line of thinking, which draws from Keynesian macroeconomics, views the cycle as a decline below some level of potential GDP. Whereas the classical view sees the economy as being a mean-reverting process that leads it back to equilibrium or potential, the Keynesian approach allows for the possibility of permanent or long-term divergences and hence argues in favour of stabilisation policies. The latter framework sees the output gap as the result of significant movements in aggregate demand in relation to a slowly moving level of aggregate supply. The emergence of an output gap is seen as being due to rigidities that delay the immediate adjustment of prices and wages. Conversely, the first line of thinking sees potential output as being 'driven by exogenous productivity shocks to aggregate supply that determine both the long-term economic growth and, to a large extent, short-term fluctuations in output over the cycle'.[7] Thus real business cycle models characterise the output gap as 'temporary disturbances caused by the adjustment of the production process to technological changes and unexpected developments on the supply side'.[8] So, as summarised in Hodrick & Prescott (1997), the classical view presents us with an equilibrium model of output while the Keynesian school is more interested in emphasising the disequilibria aspect of potential output.

[5] See for instance, Nelson & Nikolov (2001) and Stone & Wardrop (2002).
[6] European Central Bank (2011) also suggests that movements in potential output may reflect changes in the legal and economic framework, such as reforms that enhance competitiveness and reduce distortionary taxes.
[7] Scacciavillani (1999).
[8] European Central Bank (2000).

Besides its obvious use as a measure of inflationary pressures and/or the extent of under-utilisation of resources, the output gap serves other important purposes, which are more medium-term in their nature. It is used to adjust cyclically variables to reflect the levels that would prevail were the economy to operate at potential. This, for instance, can be used to ascertain the long-term sustainability of public finances.[9] The European Union has gone as far as including this concept in the Stability and Growth Pact, and uses it to assess the progress made by countries towards achieving the goal of medium-term fiscal balance.[10] Another common use of the output gap is to adjust unit labour costs in order to have a more meaningful measure of competitiveness.[11] Artis et al (2003) also show that business cycle evidence is crucial in the context of optimal currency area theory and helps indicate the optimality of monetary union. De Masi (1997) notes that the estimated trend in potential output helps determine the pace of economic growth, and so policymakers can evaluate whether they need to take steps to ensure a faster increase in the standard of living.[12]

Despite the widespread use of the concept and its many empirical applications, its use has been controversial. Plosser & William Schwert (1979) argue that a supply-oriented concept such as potential output has little operational significance. In their criticism of the Keynesian approach, they note that defined in that way potential output is not an equilibrium concept since there is no relationship between it and aggregate demand. Thus any deviation is seen as resulting from aggregate demand rather than from unexpected random shocks, giving excessive importance to demand stabilisation policies. They conclude that 'modelling potential output is an exercise with little merit, serving only to perpetuate the idea that its use as a policy guide can be justified through economic theory'.

### *1.2 Measuring potential output – empirical methods*

Potential output is unobservable and must be estimated. There is a plethora of empirical methods that have been adopted. Although some institutions have tried to gauge potential output via direct means, for example surveys of capital utilisation,[13] the preferred avenue has been that of indirect estimation techniques. These can be divided into the statistical filtering methods and the structural approaches. At the outset, however, it should be emphasised that the division between the two branches is not total, and some methods have elements of both. Moreover all approaches claim that their estimation

[9] See for instance, Giorno et al (1995).
[10] Economic Policy Committee (2001). More recently (see European Commission 2012) the EU has agreed that "annual expenditure growth not exceed the reference medium-term rate of potential GDP growth, unless the excess is matched by discretionary revenue measures".
[11] See Rotemberg & Woodford (1999).
[12] Ladiray et al (2003) includes a description of how the World Bank and the IMF look at potential output from a development economics perspective.
[13] Chagny & Dopke (2001) argue that this is too close to the engineering definition of potential output.

method is based on reasonable economic foundations and provide estimates that could be used uniformly for most analytical purposes. The literature concurs that methods should aim to 'be reasonably simple, fully transparent and easily replicable…although national specific features should be taken into account when interpreting the estimates'.[14] Among the goals most frequently put forward are that a method provides a measure of endogenous variation of economic activity, enables the evaluation of differences between changes in potential GDP and actual growth and helps assess the adequacy of economic policies.

### 1.2.1 Statistical filters – the non-structural approach

Broadly speaking, potential output in neoclassical macroeconomic thought is synonymous with the trend growth rate of actual output. Underlying this concept of potential output is the belief that left alone the economy operates at equilibrium, using resources optimally. The business cycle therefore possesses an essentially transitory nature.[15] The output series can be decomposed into a permanent component and a cyclical one that exhibits stationary behaviour in that it reverts always to the permanent level. What complicates matters is that the permanent or equilibrium level of output, however, is not stable and instead exhibits an upward trend in most economies, reflecting mainly productivity shocks.[16] Therefore methods that attempt to extricate the cyclical element must be able to identify movements in the time series that are due to the cycle and those that reflect changes in potential output.

Baxter & King (1999) argue that methods to assess the business cycle require that researchers begin by specifying characteristics of the cyclical component of the series, isolate them by simply applying moving averages and then develop adequate filters constrained to:

- extract a specified range of periodicities, and otherwise leave the properties of the extracted component unaffected;
- not alter the timing relationships between series at any frequency;
- result in a stationary time series even when applied to trending data;
- yield business cycle components that are unrelated to sample length;
- be operational, in that it does not result in too-long moving averages, with few data.

[14] Economic Policy Committee (2001).

[15] In a way, the early real business cycle school saw all fluctuations of real GDP as fluctuations in potential output, reducing considerably the importance of the concept of the business cycle.

[16] Typically over the long-term potential output is assumed to grow. However, economic literature does not exclude the possibility that it actually drops. The most obvious cases are when a country is engaged in warfare and loses part of its resources, natural disasters, and political convulsions (like the fall of communism).

The simplest method is linear de-trending. The main defect of this approach is that it implies that potential GDP growth is constant, an assumption that has little economic foundation, especially given that many economies pass through a period of rapid growth rates which then slow down as the country becomes richer. In view of this, phase average de-trending was developed, involving the estimation of a segmented linear trend, with shifts allowing for structural breaks (found by means of dummy variables). Another approach, known as peak-to-peak de-trending, suggests that maximum observed production in the past be counted as the output that is possible with full capital utilisation. The obvious disadvantage of this method is that it is not possible to determine when GDP expands quickly 'whether this is a positive trend deviation or a higher trend growth'.[17] Furthermore this method, by definition, excludes most data points of a series. Robust trend estimation, or using non-parametric regression to approximate the trend function, has also been suggested in order to surmount the problems that arise with these simple linear de-trending methods.

The most popular filtering method is the HP filter.[18] It takes the form:

$$Min(\sum_{t=1}^{T}(y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1}[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2)$$

This first term minimises the distance between the actual and the potential ($\tau_t$) value, while the second minimises the change in the trend value. Given that these two objectives contradict each other, the weight λ is used to control for the smoothness of the trend. It is typically set at 1600 for quarterly data. Its widespread use derives mainly from its simplicity, though it has other useful properties, for example it produces an output gap that is stationary even when the original series is integrated.

Despite its popularity, its use has been controversial. It is totally mechanistic and results depend crucially on the choice of λ. If λ is set at zero, the trend will be the same as the actual series, while if it is set at infinity the trend reduces to the linear form. The arbitrary choice of the smoothing parameter influences the size of potential output estimates and some studies have argued that the commonly used λs are only applicable for the normal business cycle of the United States.[19] Long run deviations from equilibrium are ruled out, and the filter assumes the business cycle lasts between 2 and 32 quarters. The method also suffers from a pronounced end-of-sample problem.[20] Baxter & King

[17] Chagny & Dopke (2001).
[18] This method is presented in Hodrick & Prescott (1997).
[19] Ibid.
[20] St-Amant & van Norden (1997) show that observations at the centre of the sample receive a 6 percent weight while the last observation accounts for 20 percent of the weight. Thus estimates of the gap for recent periods tend to change substantially as new (or revised) data are available.

(1999) find it takes data for three additional years to make sure the actual output gap makes sense.[21] Cotis et al (2005) argue that 'the sample end point weaknesses characterising HP filtering are beyond practical remedy' given that official forecasters tend to adopt a "back to average growth" forecasting approach. Another criticised feature of the HP filter is that it assumes the cycle is symmetric. In addition, Scacciavillani (1999) notes that if there are structural breaks, the use of the HP filter 'could be inappropriate since the filtering procedure may remove from the data shifts that in fact represent a change in the trend level or growth rate of potential output'.

Baxter & King (1999) instead propose a band-pass filter, which treats fluctuations longer than 6 quarters and shorter than 32 as being cyclical. Guay & St-Amant (2005), however, find fault with both the HP and the BK filters. With regard to the HP method they note that it is based on assumptions that are unlikely to be satisfied in practice:

(1) *Transitory and trend components are not correlated with each other*. This implies that the trend and cyclical components of a time series are assumed to be generated by distinct economic forces, which is often incompatible with business-cycle models.

(2) *The process $y_t$ is integrated of order 2*. Contrariwise it is usually assumed that real GDP is integrated of order 1 or stationary around a breaking trend.

(3) *The transitory component is white noise*.

(4) *The parameter controlling the smoothness of the trend component, λ, is appropriate*. But λ corresponds to the ratio of the variance of the irregular component to that of the trend component, and economic theory provides little or no guidance as to what it should be.

Guay & St-Amant (2005) show that the HP and BK filters do relatively well when applied to series having a peak in their spectrum at business-cycle frequencies. However, the authors argue that most macroeconomic time series have the typical Granger shape (i.e., they follow distinct but nonperiodic cyclical patterns, some of which seem nearly as long as the entire span of the sample). Consequently, the conditions required to obtain a good approximation of the cyclical component with these filters are rarely met in practice. More crucially, the BK filter does not give a value for the most recent quarters – those most of interest to policymakers.

### 1.2.2 Structural methods

Structural methods stand in direct contrast to statistical filters, both in terms of theoretical foundation and empirical implementation. While the filters depend crucially on the actual GDP series to determine the output gap, the structural methods derive this unobservable variable by modelling the

[21] They also criticise the usual λ's set for annual data, 100 and 400, and suggest a weight of just 10.

supply-side of the economy, to allow for long-term divergences from equilibrium. This approach tends to be favoured by policymaking institutions. This is because other methods, due to their lack of economic foundations, 'cannot underpin a comprehensive economic assessment of the outlook' and are not 'a satisfactory instrument to frame economic policy discussions or explain policy decisions to the public'.[22] They also fail to track structural changes in the economy on a timely basis and do not allow identification of the contributions of the different determinants of potential output growth. Therefore they cannot highlight economic constraints and the role of policies in enhancing potential growth.

From their inception, structural methods reflected the two main concerns of macroeconomists of the post-war period, namely demand stabilisation and growth generation. Okun (1962) defined the full employment goal 'as striving for maximum production without inflationary pressure; or, more precisely, as aiming for a point of balance between more output and greater stability, with appropriate regard for the social valuation of these two objectives'. He also noted that 'failure to use one's potential fully can influence future potential GDP' and so 'stabilising output promotes rapid growth'. Okun used unemployment as the only indicator of the output gap:

$$(U_t - \mu_t) = -\alpha\,(Y_t - \tau_t)$$

Here any deviation of unemployment from the exogenously determined non-accelerating inflation rate of unemployment (NAIRU), $\mu_t$, is used to determine the output gap on the basis of an assumed α, the degree unemployment changes over the cycle. The main assumption implicit in Okun's law is that 'whatever the influence of slack economic activity on average hours, labour force participation, and man-hour productivity, the magnitudes of all these effects are related to the unemployment rate'.[23]

This assumption, particularly problematic when there are structural labour market impediments, is one of the two main defects of this method. The other is that this method ignores the influence of the other factors of production, productivity and technological developments. This led to the formulation of the main structural method used to estimate potential GDP, namely the production function approach.[24] As its name implies, this approach involves the modelling of aggregate supply via an economy-wide production function (most frequently in Cobb-Douglas form). The validity of this method hinges on the correct specification of equilibrium employment, the capital stock and total factor productivity (TFP). The NAIRU is usually derived using a reduced-form Phillips curve, where the rate of change of nominal prices is proportional to the level of intensity of use of labour.[25] The production function

[22] Economic Policy Committee (2001). The EU Commission, for instance, decided to stop using the HP filter as its main gauge of potential output and to instead develop a simple production function.
[23] Okun (1962).
[24] See for example Denis et al (2002).
[25] Chagny & Dopke (2001).

approach allows for 'explicit accounting for growth in terms of capital, labour and TFP' and thus it is possible to 'trace the impact of various past economic disturbances on potential output, as well as to estimate the impact of current or projected disturbances on future levels of potential output'.[26]

Despite its attractiveness on a theoretical level, this approach suffers from significant practical drawbacks – the crucial one being that its data requirements are substantial. Arriving at a measure of capital stock is particularly difficult, especially if a measure of human capital is sought. Many variables, such as the NAIRU, are themselves unobservable. Furthermore real output deviates systematically from the level given by factor inputs, and the difference is usually ascribed to TFP growth. But since TFP is not directly observable, estimating its trend poses a challenge, and frequently implies a reliance on de-trending techniques – notably the HP filter.

As an alternative to production functions, some have argued for the use of structural VARs.[27] The latter involve an aggregate demand and supply model, and assume nominal shocks are neutral in the long run. Real GDP is then decomposed into the deterministic component, supply and demand shocks, with potential output being that level of output that is not affected by demand shocks. Even more recently, micro-founded dynamic stochastic general equilibrium models have been used to derive potential output – as that 'level of output prevailing under flexible prices (and wages) and perfect competition'.[28] However, their estimates of the output gap tend to be more volatile than those made using more traditional approaches, and imply 'smaller and less persistent gaps'.[29]

## *1.3 Conclusion*

In sum, the benefits of non-structural methods are that they require less information and are standardised, are simpler to implement on any time series of interest, and most force the obtained time series representing the gap to be stationary. However, using them it is impossible to disentangle the relative importance of demand and supply shocks. They rule out persistent gaps and assume cycles have more or less the same duration and are symmetric. In some cases, they also require additional judgements (on smoothing factors, the length of the cycle, etc). The structural methods, on their part, require a lot of judgement and modelling and are difficult to standardise. They are data-intensive and frequently case-specific. However they present a fuller and more theoretically rigorous approach that enables the assessment of economic policy and helps policymakers understand the causes of the current state of the economy.

[26] De Masi (1997). See Musso & Westermann (2005) for an application to the euro area.
[27] The first such example is suggested in Blanchard & Quah (1989).
[28] Vetlov et al (2011).
[29] Ibid.

Moreover, the distinction between the two strands of thought is not as clearly defined as might appear at first sight. For example, in filtering methods a production function is typically used to obtain estimates of technology shocks, and the link between inflation and the output gap is not exclusive to Keynesian economics. Thus in recent years there have been attempts to merge the two approaches – known as multivariate or semi-structural methods.[30] For instance, multivariate Beveridge-Nelson decomposition uses information from the co-movement of a number of economic time series to estimate the output gap. Another popular semi-structural approach, known as the unobserved components model,[31] uses the Kalman filter technique to derive the trend, cycle and erratic components of macroeconomic time series by imposing restrictions on the trend and cycle processes.

# 2.0 Modifying the HP filter – the case of a small open economy

When looking at a small open economy, many of the disadvantages of structural and non-structural methods are much more accentuated. Lack of data severely constrains simply relying on a production function. For example, in Malta there are no official statistics on the size of the capital stock.[32] Hence this variable needs to be estimated. Similarly, data on hours worked – crucial to determine the full-time equivalent labour force – are erratic and subject to considerable breaks.[33] The 'volatility' of factors of production then results in a relatively strange TFP growth series (as this is derived as a Solow residual). European Commission estimates of TFP for Malta, for instance, imply that from a growth rate of 7.3% in 2001, in 2002 there was a decline of -7.8%. By contrast the largest change for a bigger economy, like Germany or Italy, was of about 4 to 5 percentage points during the financial crisis.[34]

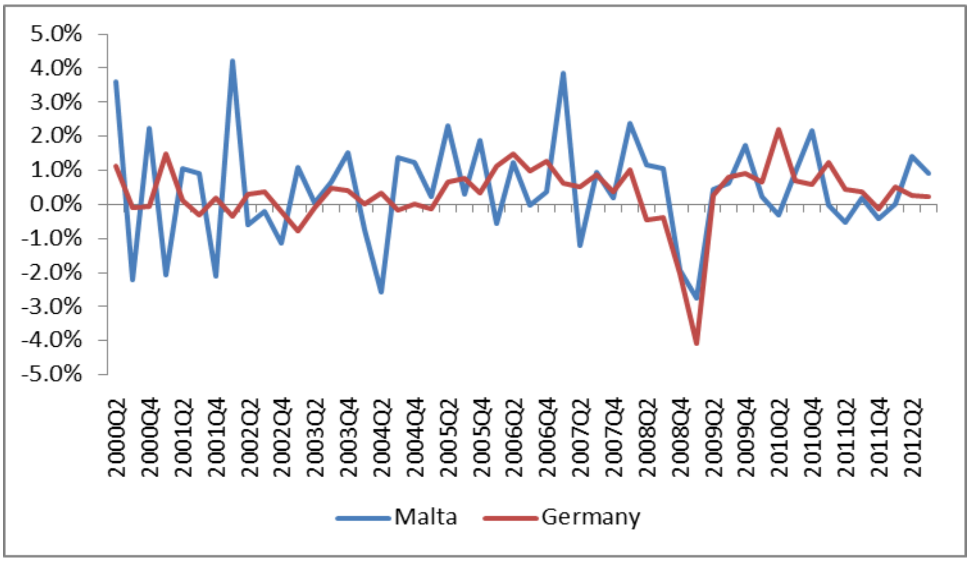
The use of semi-structural measures is also problematic. For instance, the unobserved components method tries to extract information on the output gap using observed series, such as inflation. However, these could also be subject to shocks that have little to do with changes in potential output. To give an example, between January and June 2012, overall HICP inflation in Malta rose from 1.7% to 4%. This sharp acceleration, which was reversed quickly in the following months, reflected a large temporary rise in hotel accommodation prices – rather than any acceleration in underlying activity. Similarly HICP inflation rose from 0.6% in March 2010 to 4% in December 2010, but this reflected

[30] See Ross & Ubide (2001), De Masi (1997) and Chagny & Dopke (2001) for a survey of these methods.
[31] For a comprehensive explanation of this approach, see Claus et al (2000).
[32] See D'Auria et al (2010).
[33] During the decade to 2012, the series for the average working week in Malta ranges from 41.6 hours to 34.5 hours. The same series for Germany ranges from between 37.3 hours to 34.7 hours, with a coefficient of variation half of that of the Maltese time series.
[34] For further information see https://circabc.europa.eu/faces/jsp/extension/wai/navigation/container.jsp

developments in the annual rate of change of prices of unprocessed food, which moved from a negative 7.7% to a positive 8.3%.

The repeated structural shocks and pronounced fluctuation in the GDP series for Malta also reduces the efficacy of using the standard HP filter. The latter's mechanistic nature, in fact, is even more problematic when applied to small open economies as these are more prone to have temporary but large movements in their GDP series sometimes for reasons as basic as changes in data collection. Chart 1 compares the quarter-on-quarter real GDP growth rate of Malta and that of Germany. Besides displaying very little correlation, the two series differ greatly in terms of volatility and range. For example, between the last quarter of 2001 and the first quarter of 2002, Maltese quarter-on-quarter real GDP growth moved from -2.1% to +4.2%. In the second quarter of 2007 real GDP fell by 1.2% after having risen by 3.9% in the previous quarter. By contrast, the largest change seen in the German series is the sharp recovery from a drop of 4% in the first quarter of 2009 to positive 0.2% in the following quarter. For one third of the series, the change in quarter-on-quarter growth in Malta is larger than the second-largest quarter-on-quarter change in the German GDP data.

*Chart 1: Quarter-on-Quarter real GDP growth (seasonally adjusted)*

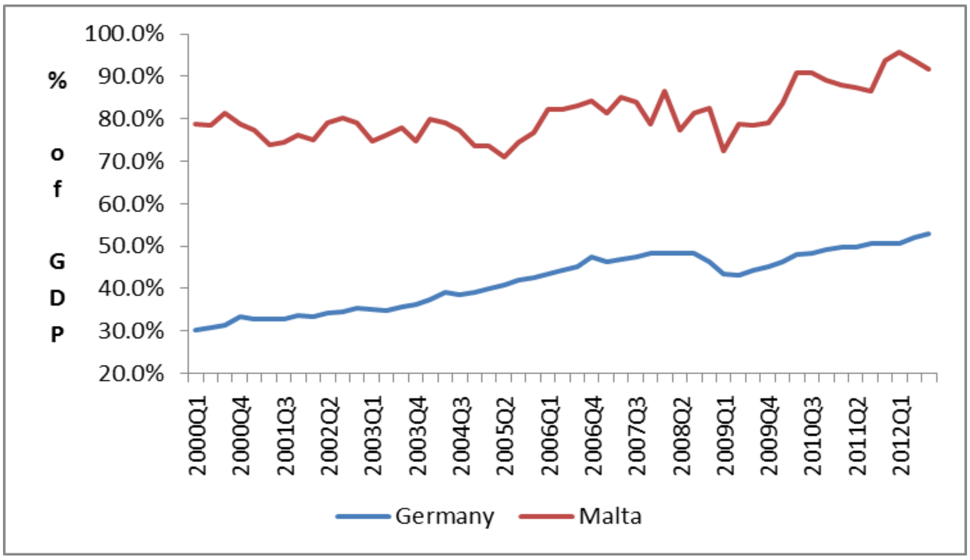



Source: Eurostat

The volatility of Malta's GDP series mostly reflects its economic characteristics. It is a very open and minute economy with quite diversified, but relatively small, export sectors. The completion of large orders or gains of significant orders, even by individual enterprises, typically results in very volatile turnover. Chart 2 shows how erratic the size of the export sector can be, even when accounting for seasonal variations. The share of exports of goods and services in German output displays a relatively smooth trend, with the exception of the immediate aftermath of the financial crisis. By contrast, in

Malta the share fluctuates widely. For instance in just a year, between the first quarter of 2009 and 2010 the share increased by 10 percentage points. Incidentally the share of exports in Malta had fallen by an equivalent amount the previous year.

*Chart 2: Exports of goods and services as per cent of seasonally-adjusted GDP*

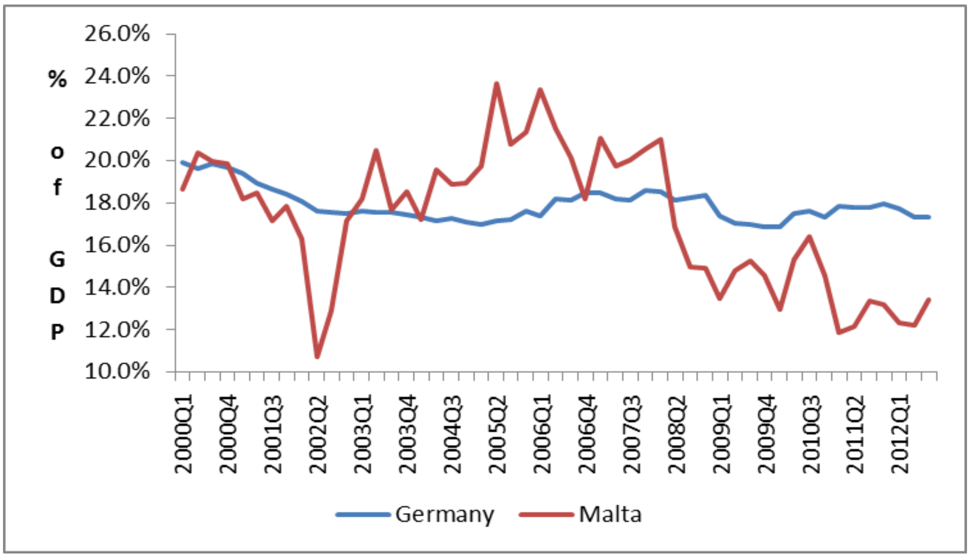



Source: Eurostat

An even starker comparison emerges when looking at the investment ratio (see Chart 3). Whereas the German series follows relatively smooth cyclical behaviour, Maltese data have no clear cycle and are full of breaks and temporary surges. In a small economy like Malta, capital projects tend to be large and one-off. For instance, the large spike in the second quarter of 2005 reflected 'to a large extent the construction of roads partly financed by funds from the EU and the Italian government'.[35] In the same vein, in the second quarter of 2002 gross fixed capital formation as a share of GDP fell to just 10.7%, as the national airline sold its airplanes and leased them back.[36] This extreme volatility is also true of government consumption (see Chart 4), where there is quite some contrast with Germany.

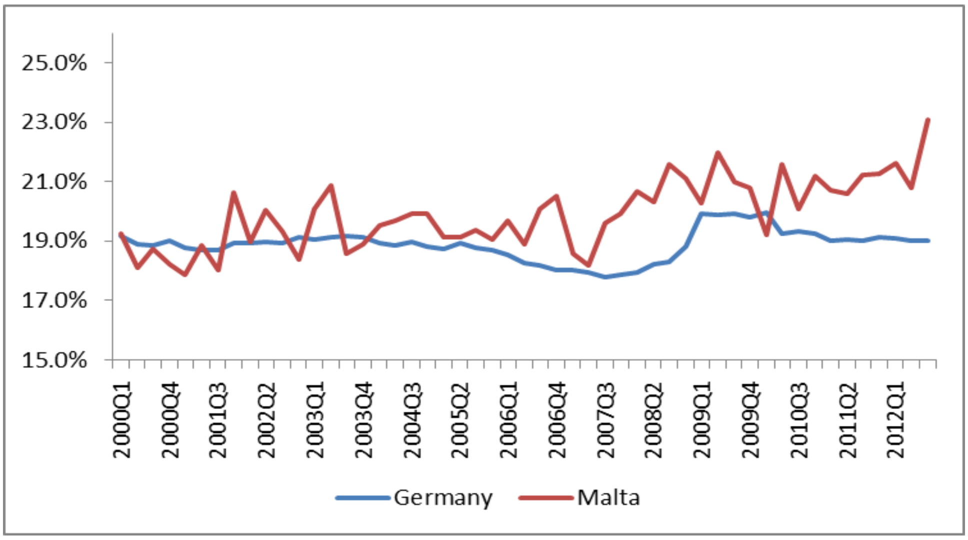
[35] Central Bank of Malta (2005).
[36] See Central Bank of Malta (2004).

*Chart 3: Gross fixed capital formation as per cent of seasonally-adjusted GDP*




Source: Eurostat

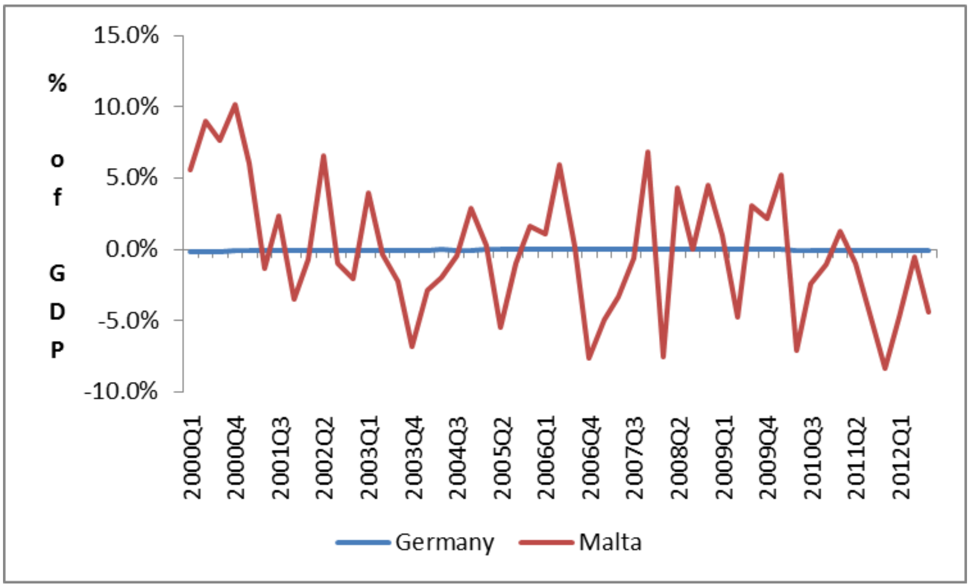
*Chart 4: General Government final consumption as per cent of seasonally-adjusted GDP*




Source: Eurostat

The volatility of Malta's GDP series is also due to data compilation methods. Inventory changes, which include a statistical discrepancy, are very large when compared to those of bigger economies, like Germany. The change in the share of inventory changes in output over two successive quarters was as much as 14% of GDP.

*Chart 5: Inventory changes as per cent of seasonally-adjusted GDP*




Source: Eurostat

Other small economies in the European Union are also characterised by very volatile national accounts data. Table 1 presents some evidence in this regard. Countries are divided into three groups, with the five large economies at one end and nine very small economies (with GDP less than 0.5% of the EU total) at the other end. The coefficient of variation of GDP data of the very small economies is nearly three times that of the larger economies. Only government consumption does not appear to have a very high coefficient of variation among very small economies. However, this does not mean that this item is not volatile, with the difference in successive quarter-on-quarter changes being very high (four-fifths of the time more than 0.6 percentage points). The range of quarter-on-quarter changes in this national accounts component is also very high in very small economies. As expected, investment and exports are a major source of volatility across all EU economies, but especially for very small economies. The latter also are characterised by a larger share of inventory changes/statistical discrepancies, which on average over the period 2000 to 2012 amounted to 0.7% of GDP, as against just 0.2% in small economies and 0.1% in the large economies.

*Table 1: Indicators of volatility in quarterly national accounts data (2000-2012) – the impact of size*

| | *Very small countries (with own GDP less than 0.5% of total EU GDP)* | *Small countries (with own GDP between 0.5% and 5% of total EU GDP)* | *Large countries (with own GDP greater than 5% of total EU GDP)* |
|---|---|---|---|
| ***Coefficient of variation*** | | | |
| GDP | 13.7% | 7.5% | 4.7% |
| Private consumption | 15.0% | 6.5% | 4.1% |
| Gov. consumption | 6.2% | 7.3% | 6.4% |
| Exports | 25.3% | 15.0% | 13.5% |
| Investment | 16.1% | 8.4% | 6.3% |
| ***Range of quarter-on-quarter changes*** | | | |
| GDP | 8.4 percentage points | 3.7 percentage points | 3.7 percentage points |
| Private consumption | 5.9 percentage points | 1.9 percentage points | 1.7 percentage points |
| Gov. consumption | 14.5 percentage points | 3.1 percentage points | 1.8 percentage points |
| Exports | 13.1 percentage points | 9.9 percentage points | 13.7 percentage points |
| Investment | 14.6 percentage points | 8.6 percentage points | 9.4 percentage points |
| ***Odds of successive quarter-on-quarter changes varying by more than 0.6 p. p.*** | | | |
| GDP | 0.27 | 0.18 | 0.12 |
| Private consumption | 0.31 | 0.12 | 0.10 |
| Gov. consumption | 0.80 | 0.29 | 0.14 |
| Exports | 0.76 | 0.67 | 0.76 |
| Investment | 0.78 | 0.65 | 0.59 |
| ***Mean value of Inventory changes/Statistical discrepancy (% of GDP)*** | 0.7% | 0.3% | 0.1% |

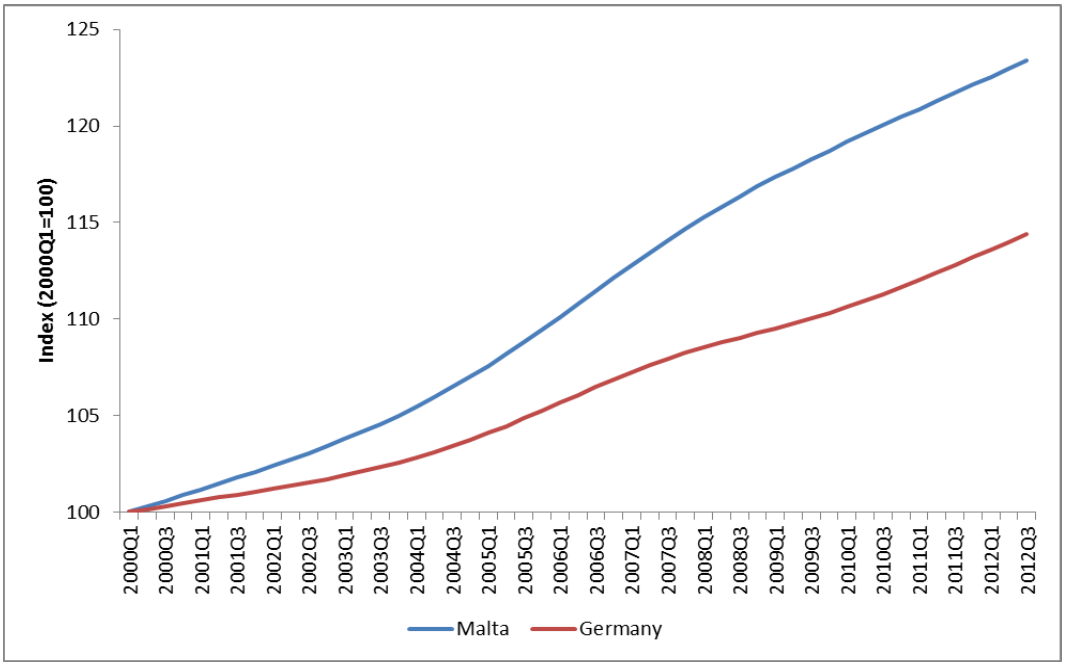
Source: Own analysis using Eurostat GDP seasonally-adjusted quarterly data

All this suggests that the best way to evaluate potential output in a small open economy setting is to try to integrate the structural and non-structural approaches. Given its widespread use and the frequent reliance on this method in production functions, the following sections suggest ways how the HP filter could be modified, or used, in a way to reduce its mechanistic nature and make it more appropriate for use in series that are subject to frequent and pronounced shocks.

### *2.1 A suggested alternative application of the standard HP filter*

Potential output is not something which is expected to drop, except in exceptional cases like natural disasters, massive emigration or wars.[37] There is increasing evidence that growth in potential output is affected by severe financial crises or deep recessions[38], but that there is very little evidence that a recession would reduce potential output permanently from its pre-crisis level – as this would imply that the recession would destroy factors of production and/or the efficacy of using them (e.g. after being unemployed for a long period, workers would be less productive when they are reemployed). Running an HP filter on quarterly GDP data for Malta and Germany results, in line with a priori expectations, constantly improving potential output levels, as can be seen in Chart 6.

*Chart 6: Index of Potential Output (HP filter on seasonally-adjusted quarterly GDP data)*



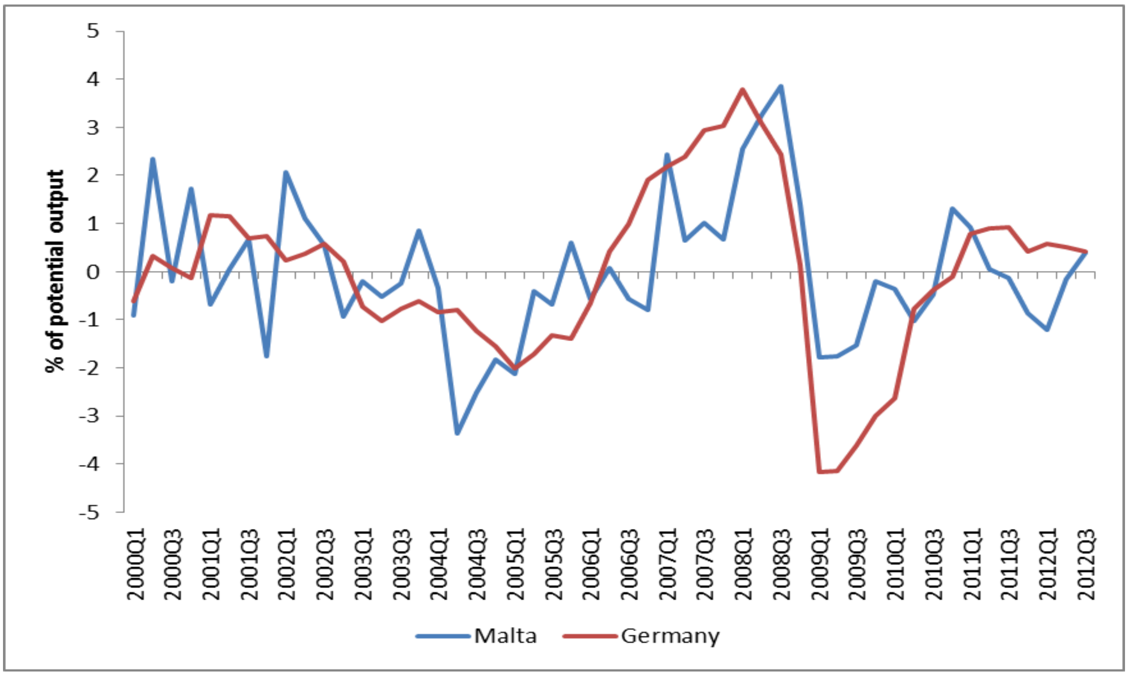

Source: Own calculations using Eurostat data

[37] Chagny & Dopke (2001) raise this point, for instance.

[38] Haltmaier (2012) finds, for instance, that 'the Great Recession might have resulted in declines in trend output growth averaging about 3 percent for the advanced economies, but appear to have had little effect on emerging market trend growth'. Similar results are found in European Commission (2009).

However, due to the large element of noise in the data, applying the standard HP filter on the GDP series of a small open economy gives the impression that the output gap is very volatile.[39] For instance when applied to the Maltese quarterly real GDP series,[40] the HP filter yielded a output gap series that ranged between –3.4% and +3.8% of potential GDP. This is not that different from the range found looking at German data (i.e. -4.1% to +3.8%). However, while the Maltese series was stationary (as by definition of equilibrium output), as can be seen from Chart 7 it was erratic and far less smooth, and displayed no clear cyclical behaviour. For instance, looking at 2000, we move from an output gap of 0.9% in the first quarter to an output surplus of 2.3% in the second, an output gap of 0.2% in the third and an output surplus of 1.7% in the last quarter of the year. The Maltese output gap series changed sign 19 times (out of a possible 50 times) while the German series did so only 7 times. The relevance of such a measure of 'equilibrium' is clearly limited and it cannot serve as a good guide for decisions on economic policy. If a measure shows that the economy is always in substantial disequilibrium, this would raise the question of whether it actually makes sense to attempt to assess the equilibrium output of an economy. By the time a policy is decided upon and starts taking effect, it would not be desirable anymore.

*Chart 7: Output Gap (HP filter on seasonally-adjusted quarterly GDP data)*




Source: Own calculations using Eurostat data

[39] Canova (1998) points out that filters assume 'that the irregular (high frequency) fluctuations play little role'.
[40] To reduce the end-of-sample problem, the GDP series for Malta and Germany were extended forwards for four years beyond the historical sample by means of a simple ARMA(1,1) forecast.

The variability of the HP-filtered output gap measure for Malta can, however, be reduced.[41] A graphical analysis of the GDP time series helped determine an upper and a lower GDP data sequence, consisting of the minima and maxima of the movements of the series. These points were then interpolated linearly. An HP filter was then applied separately on the upper and lower data sequence. Then for some weight, $\theta$ (with a value between 0 and 1), the output series was defined as:

$$\theta Y_L + (1 - \theta) Y_H$$

where $Y_L$ stands for the trend of the lower bound of the series while $Y_H$ is the trend fitted to the upper bound. The value of $\theta$ needs to be set on the basis of an assessment as to where the equilibrium lies. It would be closer to 1 the more one believes that equilibrium lies close to the trend set by the lower bound of the time series, and vice versa. Conversely given that potential output is generally taken to mean 'maximum' output, one could argue that the bias for $\theta$ should be closer to 0. The setting of $\theta$ introduces an element of judgement, which inevitably implies that the method is not easy to standardise across countries.

Chart 8 shows the upper and lower bound series of Malta's seasonally-adjusted quarterly real GDP series. The upper bound takes all those GDP readings which are underpinned by a quarter-on-quarter increase, while the lower bound takes the readings resulting from a quarter-on-quarter decline. The points between successive actual values of these series were interpolated linearly as there was very little distance in time between them. Both series are considerably smoother than the actual GDP series, especially in the first two to three years of the sample. While this method might be criticised as not using the actual GDP series, note that at any point in time either the upper bound or the lower bound series would be equal to the actual GDP series. The method, however, does depend crucially on the identification of the upper and lower bounds of a series. This can be problematic when applied to a series without a pronounced cycle or one that exhibits some form of upward trend. There is also the added problem that it is very hard to determine whether the most recent data constitute effectively maxima or minima, as this assessment could be overturned once data for the future are recorded. So this alternative use of the HP filter would be prone to revisions, especially for the most recent points.

[41] Other small economies also face the issue of having very volatile quarterly output gap series. Grech (2013) applies a number of methods to address this issue for the six smallest Euro area economies.

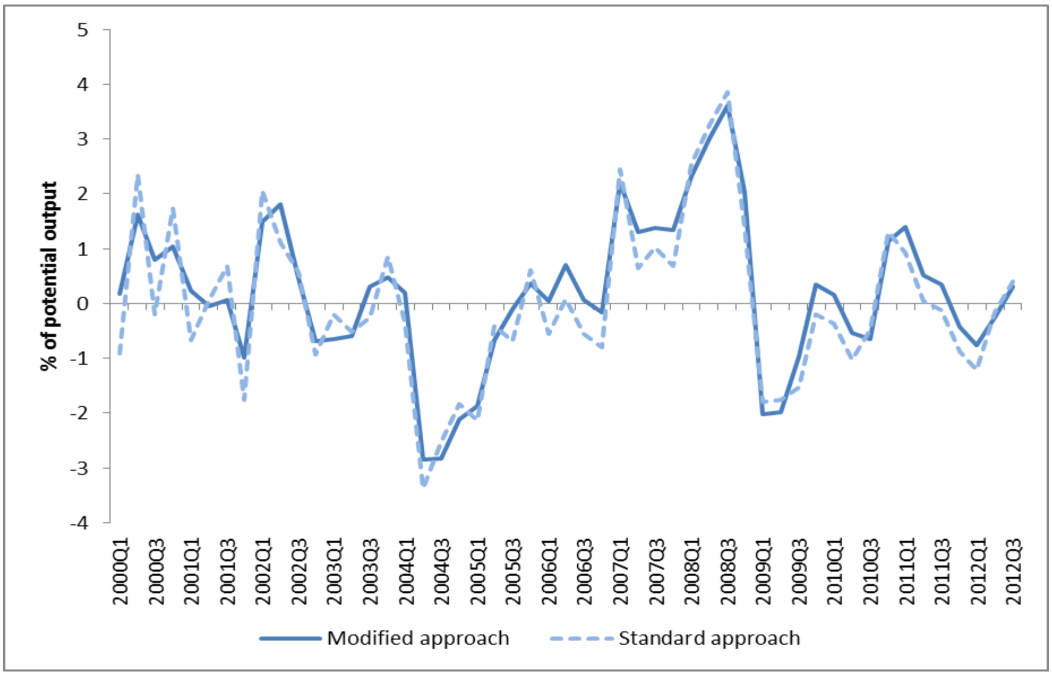
*Chart 8: Lower and upper bounds of seasonally-adjusted quarterly GDP data for Malta*


Source: Own calculations using Eurostat data

Having settled on the upper and lower bounds of the series, the next step is to determine the value of $\theta$. As has been argued earlier, Malta's GDP is frequently boosted by one-off events such as large capital projects and large export orders. Thus $\theta$ was set as close as possible to 1, namely 0.99, as the lower bound series is less affected by these events. Adopting a constant $\theta$ does not necessarily affect the variability of the output gap estimates. It just allows one to have a lower potential output series.

Varying $\theta$ over time can, however, help reduce the variability of the output gap. The algorithm used to determine this variation was based on observing the changes in the standard HP filter derived output gap for Malta and comparing them with those of a larger (and more stable) economy (in this case, Germany). Whenever the change in the standard HP filter output gap for Malta was substantially larger than that in Germany, the $\theta$ was changed to 0.01, so that the upper bound series is given the highest weight. In this way, potential output is allowed to accelerate when there is a spike in growth, but not be dragged downwards by abrupt downturns. To help smooth this effect, in the quarter following a change $\theta$ is set at 0.35, before then reverting to the standard 0.99. In all, this shift needed to be done for 13 quarters, out of a total of 52 quarters.

*Chart 9: Output gap resulting from using standard HP filter and the modified approach*




Source: Own calculations using Eurostat data

As can be seen from Chart 9, the output gap series resulting from the modified approach is significantly smoother than that derived from the standard HP filter. Earlier we had seen that the standard HP filter suggests that during 2000, Malta had an output gap of 0.9% in the first quarter, a surplus of 2.3% in the second, then a gap of 0.2% in the third, followed by a surplus of 1.7% in the last quarter. Using the modified approach, in the same year we now have a surplus of 0.2%, 1.6%, 0.8% and 1% in the respective four quarters. The number of times the output gap changes sign is also significantly lower, while the series' standard deviation is smaller. The correlation of the Maltese and German output gap series also improves to reach nearly 0.7 (an improvement of about a tenth). While Malta still appears to have a more volatile series and has less well-defined cycles, this modified approach makes the series more in line with those of larger economies. Refining the algorithm to introduce a more dynamic setting of $\theta$ could help make the Maltese series smoother, but would run the risk of making the implied potential output growth too erratic. The chosen algorithm resulted in the mean value of the output gap being very close to 0 over the period considered.

Another possible use of this approach is to allow for the possibility of long-term deviations from equilibrium. This is particularly useful if the series one has to work with is very short – a common occurrence when working with data for small open economies. In this case, one can set the factor $\theta$ on the basis of considerations from other data (e.g. indications from sentiment indicators or capacity utilisation rates) or by looking at developments in related economies for which the data series is longer and less affected by one-off events. If one sets $\theta$ close to 1, the series for the gap has a positive mean value and the reverse if one sets $\theta$ at, or close to 0.

Of course, the extent of the deviation can be controlled by applying the factor differently in different time periods. For instance, in an initial period where the researcher believes there was under-employment of resources, $\theta$ would be set closer to 0 and then this would be changed gradually to become closer to 1. The other thing to consider is that the extent to which $\theta$ would need to be modified depends crucially on the nature of the upper and lower bound series. If these are similar, as is typically the case in larger economies, the factor will not play an important role. When there is a noticeable gap, the setting of $\theta$ would require some thought.

## *2.2 A suggested modification to the standard HP filter*

Another more radical solution would be to modify the standard HP filter rather than just using it differently. Owing to its shortcomings, there have been some such attempts. In their survey on potential output estimation methods, Chagny & Dopke (2001) describe one such attempt – the multivariate HP filter.[42] The latter attempts to give the filter an economic basis by means of three structural equations. This method, in fact, involves defining an augmented Phillips curve with inflation depending on expected inflation and the current and lagged output gap. An Okun's law relation is then specified, with current unemployment depending on an exogenously determined NAIRU and the current and lagged output gap. Then one calculates capacity utilisation and compares it vis-à-vis its optimal level. A loss function is then set up that minimises the standard HP filter and the residuals of these three equations. The problem the authors note in this approach is that it is hard to judge the weight to set to the structural equations in the loss function.

Ross & Ubide (2001) review another modification known as HP-ARIMA. The latter aims to minimise the end-of-sample problem by forecasting and backcasting the series with an ARIMA model. Since the choice of the smoothing parameter, $\lambda$, amounts to identifying the allocation of variations in output to trend and to cyclical components, they suggest calculating $\lambda$ in a cross-country setting so as to equalise the volatility of the trend across countries, using one country as the benchmark.[43] They propose two methods; allowing for a larger variability of the growth rate in countries with a more volatile component, or else assuming similar economic structures between the benchmark and the comparator country.

[42] See Areosa (2008) for a somewhat similar modification of the HP filter.
[43] In a way, this is how the factor $\theta$ was set in the previous section.

Dennis & Razzak (1999) made another important contribution, basing their modification to the standard HP filter on the fact that λ can be interpreted as a function of the ratio of the variances of aggregate demand and supply. They note that setting a constant λ implicitly assumes that the relative variances of demand and supply disturbances of output are time-invariant. Applying their thinking to the case of New Zealand, a small open economy, they suggest that keeping λ constant does not make sense. New Zealand's GDP has undergone a number of significant structural shocks, some of which were policy-driven, and, hence, they argue in favour of allowing λ to vary with time to reflect changes in the variation of aggregate demand and supply.[44] The idea of having a time-variant λ is attractive as the effect of structural shocks and one-off factors could be reduced and, thus, the potential output series would give a fairer picture of the equilibrium of the economy. However, there is an alternative way to arrive at a similar result via a more fundamental modification of the standard HP filter, intended to give researchers leeway to reduce the mechanistic nature of the filter and adjust it for underlying characteristics of a particular series.

Consider the following modification of the standard HP filter:

$$Min(\sum_{t=1}^{T} w_t( y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t)-(\tau_t - \tau_{t-1})]^2)$$

The second part of the equation (including $\lambda$), which smoothens the growth in potential output, has not been changed. Instead the first part of the equation now includes an additional term, $w_t$. This is a generalisation of the standard HP filter, which is the case when $w_t$ is 1. The weight $w_t$ is the means by which the researcher would introduce judgement in the HP filter and modify it according to a series' particular characteristics.

The choice of this weight would reflect two considerations, namely:

a) $w_t$ is set lower, the more $y_t$ is below the previous high *y* value, with maximum $w_t$ = 1 when $y_t$ is higher or equal to the previous high. More formally, if at time *t* the previous high *y* is $h_t$, then:

$$w_t = \frac{1}{1 + \mu\{\max(h_t - y_t, 0)\}}$$

Through this modification, a researcher would be able to control the extent to which potential output falls in level by means of the value of the factor $\mu$. For example if there is a gap of €100 million (about 7% of Malta's average quarterly GDP) from the previous high level of GDP, with a $\mu$ of 0.001,

[44] Again, the approach of setting a time-varying factor $\theta$ in the previous section follows a similar line of thinking.

you would have a $w_t$ of 0.9. A gap of €500 million (about a third of Malta's average quarterly GDP) would result in a $w_t$ of 0.67. A higher $\mu$ would result in less weight being given to quarters when GDP is much lower than the previous (local) maximum. The logic behind this is that drops in the level of potential output should be relatively rare, and occur only when resources or knowledge are destroyed. Statistical filters should have a bias against allowing for a decline in equilibrium output. There are cases when the application of the standard methods could lead to declines in potential output when there is no contraction in an economy's capacity. Small open economies, due to their size, frequently have investment figures that fluctuate substantially, leading to variability in GDP, as capital projects tend to be large and one-off. Returning to an example given earlier, the large drop in gross fixed capital formation in Malta in 2002 simply reflected the fact that the national airline sold its airplanes and leased them back. This accounting move did not affect the productive capacity of the Maltese economy but the application of the standard HP filter would show a decline in potential output. The suggested modification allows the fine tuning of the filter to take into account one-off factors.

b) $w_t$ is set lower the greater the time difference between $y_t$ and $h_t$. Letting $p_t$ denote this difference:

$$w_t = \frac{1}{1 + \varphi\,\{\max(p_t, 0)\}}$$

The main idea behind this modification is to introduce the possibility that there may be long-term deviations from equilibrium. This is done by means of the factor $\varphi$. For instance, if 10 quarters have passed since the last peak of GDP, with $\varphi$ at 0.05, $w_t$ would be 0.67. If the latest peak occurred two quarters ago, then with the same factor, $w_t$ would be 0.91 Therefore for a decline in output to be allowed to affect the level of potential output; the decline must persist for some specified time. In the standard HP filter, declines in output automatically result in lower potential GDP, thus eliminating the possibility of long disequilibria. With this modification, the researcher has the discretion to allow for longer periods when output is below potential.

Uniting these two concepts, the weight $w_t$ in the first part of the HP filter would be determined as:

$$w_t = \frac{1}{1 + \mu\,\{\max(h_t - y_t, 0)\}} \times \frac{1}{1 + \varphi\,\{\max(p_t, 0)\}}$$

This new weight has two parts: the first affecting the impact of temporary shocks on a series, and the second allowing for the Keynesian concept that there may be long-term deviations from equilibrium. The factors, $\mu$ and $\varphi$, would be chosen by the researcher, on the basis of judgement of the particular characteristics of a series. The closer they are to 0, the more the underlying series is allowed to determine potential output.[45] On the statistical side, the issues to keep in mind are the variability of the series and the possibility that downward movements in a series may be more serious than upturns. On the economic side, one needs to consider that inflation, unemployment and other such variables of interest to policymakers tend to respond more to upward movements in the GDP series than to downward movements. Not only are prices notoriously sticky downwards, but also the presence of labour hoarding in small economies typically means that unemployment takes time to rise after an economic downturn. These factors should be taken into consideration when setting $\mu$ and $\varphi$. The researcher would need to look at micro-studies or other evidence on a number of factors, such as the level of education of the workforce, the amount and quality of the capital stock and other measures of potential resources, past evidence of hysteresis effects, when trying to determine the value of $w_t$. The factors $\mu$ and $\varphi$ can also be interpreted as indicating the rate of adaptability of factors of production to economic shocks and the rate of recovery of an economy from downturns. Studies of past recessions or cross-country evidence on cyclical fluctuations would be very helpful to set these factors.

Another thing to consider is whether to allow $\mu$ and $\varphi$ to vary over time. When tested on Maltese data, applying constant factors resulted in an output gap series that had a slightly higher standard deviation than that derived using the standard approach. This possibly reflects the noise present in Maltese data.[46] Thus the approach taken was to apply a $\mu$ and $\varphi$ of 0.00125 for most of the period, except for those quarters where the change in the standard HP filter output gap for Malta was substantially larger than in Germany. In these quarters the sign of $\mu$ and $\varphi$ was changed, to try to address potential noise in the Maltese series.

Note that the solution of the generalised HP filter differs slightly from that of the standard HP filter, as one needs to consider the influence of $w_t$. The solution of the minimisation problem in the standard filter[47] is:

$$\tau = \left(I_T + \lambda K^{'}K\right)^{-1} X$$

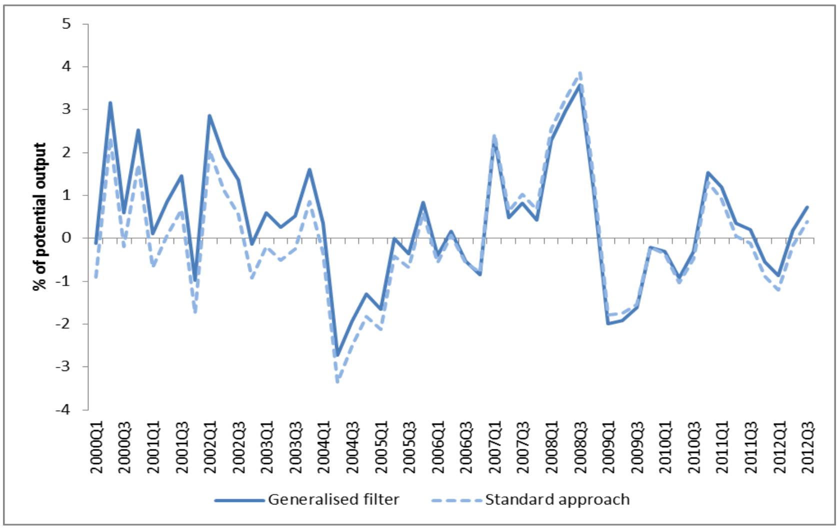
where $\tau$ is a vector with the values of the filtered series, $I_T$ is the identity matrix with dimension $T$, $X$ is a vector with the readings of the series being filtered, $\lambda$ is the smoothing parameter, and

[45] Of course they need to be less than 1, or else they would have too strong an impact. In the case of $\mu$ the value would need to be quite small, as the difference between current GDP and its previous peak could be quite high. Alternatively one could define current GDP and its previous peak in logs or in terms of the series' mean value.
[46] In fact, when this approach was applied to the German GDP data, there was no effective difference between time-varying and time-invariant parameters.
[47] One takes the derivative of the minimisation problem with respect to $\tau$, re-arranges and converts to matrices.

$$K = \begin{pmatrix} 1 & -2 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & -2 & 1 \end{pmatrix}$$

On the other hand, in the generalised filter, $X$ is a vector with the readings of the series being filtered multiplied by their respective $w_t$, derived on the basis of the formula described above.

*Chart 10: Output gap resulting from using standard HP filter and the generalised filter*




Source: Own calculations using Eurostat data

Chart 10 shows the output gap resulting from the application of this generalised HP filter to Maltese quarterly GDP data, using time-varying parameters. Compared to the output gap series derived using the modified approach described in section 2.1, the results are less smooth, particularly for the start of the period. However, similar to the modified approach, the generalised filter results in considerably less changes in sign of the output gap series, particularly within the same year, and has a lower standard deviation. Further improvements of the algorithm used to set the parameters determining $w_t$ could result in a smoother series, but the scope of this exercise was just to outline the approach.

## 2.3 Conclusion

In view of the problems encountered with the use of the standard HP filter on data for small open economies, this section has suggested some ways to try to make the filter less mechanistic, incorporate some structural features and allow for some element of judgement. The first suggestion involves an innovative application of the HP filter to the upper and lower bounds of the GDP series, resulting in a smoother output gap series with the possibility of long-term deviations from equilibrium. The second suggestion is a modification of the HP filter to give researchers discretion on the impact exerted by structural or temporary shocks, while allowing for the possibility of having lengthy periods of disequilibria. This modification can be called a ‘generalisation’ of the standard HP filter, as the latter is nested within the broader framework allowed by the modified filter. Both adjustments could lead to a better estimation of potential output for small open economies.

## Conclusion

The concept of potential output constitutes one of the most fundamental ideas of macroeconomics. It encapsulates the notion that there is an equilibrium or optimal level of production. Naturally, policymakers strive to establish an economic environment consistent with the operation of the economy at this level. Knowing the extent of the output gap, or the deviation from this equilibrium, is therefore very important for those who believe that the equilibrating forces in a market economy need to be sustained by discretionary policy. However, even those who are less trusting in the efficacy of discretionary policy typically concur that policymakers can affect the growth of potential output with their policies on education, taxation and infrastructure among other things.

Arriving at a reliable estimate of potential output is therefore a goal that is of prime interest to policymakers, both on a national and on a supranational setting. This interest has fuelled significant research and a plethora of methods have been developed over the years. This paper has sought to outline why the most commonly used approaches can be problematic when applied to the special case of Malta, a very small and open economy. Lack of data, and erratic developments in investment and labour demand, complicate the construction of a production function based on quarterly data. Malta’s quarterly GDP series, even when seasonally adjusted, does not lend itself well to the statistical properties underpinning filtering methods. The series is short, has large fluctuations and recurrent structural breaks. Moreover, the aggregate series is underpinned by even more volatile data on the components of GDP, particularly inventories. This decreases the reliability of the standard HP filter, which when not used as the direct measure of potential output itself is frequently an important determinant of certain components of the production function approach. When applied to Malta, the filter exhibits very pronounced changes in the output gap that are inconsistent with the theoretical idea of equilibrium. The filter is affected significantly by shocks to data, which in turn reflect the small size of the Maltese economy rather than actual changes in potential output.

On the basis of these findings, this paper put forward two suggestions. The first involves an innovative application of the standard filter, whereby the upper and lower bounds of a series are defined and the equilibrium level is determined as a weighted average of the HP filter applied separately on these bounds. This can result in a smoother output gap series with the possibility of long-term deviations from equilibrium. The second suggestion involves a significant rethinking of the standard HP filter to integrate structural features. The modified or generalised HP filter would allow researchers to set limits on the impact exerted by structural or temporary shocks and to allow for the possibility of having lengthy periods of disequilibria.

These suggestions can be criticised as granting too much discretion to researchers and thus doing away with one of the main benefits of the HP filter, namely the lack of judgement needed to apply it. However, the special characteristics shown by the output series of small open economies make this benefit appear to be more of a drawback. Rather than ignoring the properties of the series being studied, these suggested modifications allow researchers to apply their judgement and knowledge on a particular economy to arrive at a measure of equilibrium that is more based on sound economic reasoning. Though this would probably lead to various estimates of the output gap being made for the same economy, a healthy debate between economists on the adequate parameters to be set is much to be preferred to a blind adherence to a standardised filtering method. Given the importance of gauging the level of potential output properly, it is better to have a number of partially correct opinions rather than one incorrect result.

## Bibliography


Alesina A. & E. Spolaore (2003), “The size of nations”, MIT Press.

Areosa M. (2008), “Combining Hodrick-Prescott filtering with a production function approach to estimate output gap”, Working Paper 172, Banco Central do Brasil.

Artis M., M. Marcellino & T. Proietti (2003), “Dating the Euro Area business cycle”, Working Paper N 237, IGIER, Universita’ Bocconi (Italy).

Baxter M. & R.G. King (1999), “Measuring business cycles – Approximate band-pass filters for economic time series”, The Review of Economics and Statistics, Vol. 81, Issue 4, p. 575-593.

Blanchard O. J. & D. Quah (1988), “The dynamic effect of aggregate demand and supply disturbances”, American Economic Review Vol. 79, p. 655-673.

Canova F. (1998), “De-trending and business cycle facts”, Journal of Monetary Economics, Vol. 41, p. 475-512.

Central Bank of Malta (2004), “Annual Report”.

Central Bank of Malta (2005), “Annual Report”.

Chagny O. & J. Dopke (2001), “Measures of output gap in the Euro-zone: An empirical assessment of selected methods”, Working Paper No.1053, Kiel Institute of World Economics.

Claus I., P. Conway & A. Scott (2000), “The output gap: measurement, comparisons and assessment”, Research Paper No.44, Reserve Bank of New Zealand.

Cotis J-P, J. Elmeskov & A. Mourougane (2005), “Estimates of potential output: Benefits and pitfalls from a policy perspective”, in Reichlin L. (ed.), The Euro Area business cycle: Stylized facts and measurement issues, London: CEPR.

D’Auria F., C. Denis, K. Havik, K. McMorrow, C. Planas, R. Raciborski, W. Roger & A. Rossi (2010), “The production function methodology for calculating potential growth rates and output gaps”, European Economy, Economic Papers 420, European Commission.

De Masi P. R. (1997), “IMF estimates of potential output: Theory and practice”, Working Paper WP/97/177, International Monetary Fund.

Denis C, K. Mc Morrow & W. Roger (2002), “Production function approach to calculating potential growth and output gaps – estimates for the EU Member States and the US”, Economic Paper No 176, European Commission.

Dennis R. & W.A. Razzak (1999), “The output gap using the Hodrick-Prescott filter with a non-constant smoothing parameter: an application to New Zealand”, Working Paper, Reserve Bank of New Zealand.

Economic Policy Committee (2001), “Report on potential output and the output gap”, Report to the Directorate Generale Economic and Financial Affairs, European Commission.

European Central Bank (2000), “Potential output growth and output gaps: concepts, uses and estimates”, Monthly Bulletin: October, European Central Bank.

European Central Bank (2011), “Trends in potential output”, Monthly Bulletin: January, European Central Bank.

European Commission (2009), “Impact of the current economic and financial crisis on potential output”, European Economy, Occasional Paper 49.

European Commission (2012), “Specifications on the implementation of the Stability and Growth Pact and Guidelines on the format and content of Stability and Convergence Programmes”, Code of Conduct.

Giorno C., P. Richardson, D. Roseveare, & P. van den Noord (1995), “Estimating potential output, output gaps and structural budget balances”, Working Paper No.152, Organisation for Economic Co-operation and Development.

Grech A. G. (2013), “Adapting the Hodrick-Prescott filter for very small open economies”, International Journal of Economics and Finance, Vol. 5, no. 8, 39-53.

Guay A. & P St-Amant (2005), “Do the Hodrick-Prescott and Baxter-King filters provide a good approximation of business cycles?”,Annales d’Economie et de Statistique, No.77, ENSAE.

Haltmaier J. (2012), “Do recessions affect potential output?”, International Finance Discussion Paper No 1066, Board of Governors of the Federal Reserve System.

Hodrick R.J. & E.C. Prescott (1997), “Post-war US business cycles: An empirical investigation”, Journal of Money, Credit and Banking, Vol. 29, No. 1, 1-16.

Ladiray D., G.L. Mazzi & F. Sartori (2003), “Statistical methods for potential output estimation and cycle extraction”, Eurostat.

Musso A. & T. Westermann (2005), “Assessing potential output growth in the euro area: A growth accounting perspective, Occasional Paper No. 22, European Central Bank.

Nelson E. & K. Nikolov (2001), “UK inflation in the 1970s and 1980s: the role of output gap mismeasurement”, Working Paper No.148, Bank of England.

Okun, A. (1962), “Potential GDP: Its Measurement and Significance”, American Statistical Association, Proceedings of the Business and Economic Statistics Section: 98-103. Washington.

Plosser C. & G. William Schwert (1979), “Potential GNP: Its measurement and significance – a dissenting opinion”, Carnegie-Rochester Conference Series on Public Policy, Supplement to the Journal of Monetary Economics 10.

Ross K. & A. Ubide (2001), “Mind the gap: What is the best measure of slack in the Euro Area?”, Working Paper WP/01/203, International Monetary Fund.

Rotemberg J.J. & M. Woodford (1999), “The cyclical behaviour of prices and costs”, in ‘Handbook of Macroeconomics, Volume 1, Edited by J.B. Taylor & M. Woodford, Elsevier.

Scacciavillani F. & P. Swagel (1999), “Measures of potential output: An application to Israel”, Working Paper WP/99/96, International Monetary Fund.

St-Amant P. & S. van Norden (1997), “Measurement of the output gap: A discussion of recent research at the Bank of Canada”, Technical Report No.79, Bank of Canada.

Stone A. & S. Wardrop (2002), “Real-time national accounts data”, Research Discussion Paper 2002-05, Reserve Bank of Australia.

Vetlov I., T. Hledik, M. Jonsson, H. Kucsera & M. Pisani (2011), "Potential output in DSGE models", Working Paper No. 1351, European Central Bank.