Time variation, asymmetry and threshold effects in Malta’s Phillips curve

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Abstract

This paper estimates a Phillips curve for Malta using data since the 1960s and presents evidence that the relationship has flattened over time, implying that the link between inflation and economic activity has weakened. Further analysis finds that this phenomenon was driven by downward price stickiness, whereby the responsiveness of price inflation is limited during economic slowdowns; however the Phillips curve was and remains alive during economic booms. Other threshold effects are also present; with only statistically large shocks having an effect during a boom. This study also finds a strong increase in the importance of import price shocks in driving consumer price inflation in the Maltese economy. The estimated variance of shocks to inflation was high in the 1980s, but has fallen greatly since then.

Estimates of the Wage Phillips curve show a strong link between economic activity and wage growth. This channel operates mainly through slower wage growth during recessions, as periods of strong economic growth were not associated with faster wage growth. Similar threshold effects also exist; this channel only operates when economic slack is high. A number of structural changes in the Maltese economy, notably EU accession and favourable labour market developments may explain the stability in wage growth and some of the findings.

JEL Classification Codes: C11, C32, E31, E32, O11

Keywords: Inflation, Phillips curve, time-varying parameters
1 Introduction

Understanding inflation dynamics has become particularly important in view of the low inflation regime now prevailing and because the traditional relationship between slack in the economy and inflation seems to have weakened significantly in some countries.

If confirmed, the flattening of the Phillips curve would be relevant for monetary policy because that relationship was the traditional linchpin of the transmission mechanism that gave central banks control of inflation. The subsequent focus on the role of expectations and their management in the toolkit of monetary policy reduced but did not eliminate the relevance of the traditional mechanism.

—Vítor Constâncio
ECB Vice-President

Central banks have striven to earn credibility in their quest to control consumer price inflation by, inter alia, improving their communication through the announcement of a preferred rate for inflation at which the economy can operate. For example, monetary policy in the euro area is conducted with the primary objective of keeping inflation “below, but close to, 2% in the medium term” (ECB, 2001). In the pursuit of analysing economic developments, econometric models help shape views about the current and medium-term outlook for economic activity and inflationary pressures.

In the wake of the financial crises of 2008 and the ensuing Great Recession, which was largely unpredictable even by state-of-the-art models, research has been directed at studying additional important channels through which shocks propagate. During the same time it was observed that models that historically enjoyed a good track record at forecasting inflation tended to perform badly, predicting a more significant drop in inflation than what materialised. This was termed the period of the ‘missing deflation’ (Ball and Mazumder, 2011; Stock, 2011; Ball and Mazumder, 2015).3 As a result another strand of research focused on analysing whether structural changes in otherwise standard macroeconomic relationships could explain this anomaly in advanced economies such as the euro area.

Economists believe that in the short run inflation moves in line with economic conditions. This relationship, known as the Phillips curve, traces its origins to an empirical exercise showing the existence of a negative relationship between nominal wage growth and unemployment in the United Kingdom, which A.W. Phillips published in 1958 (Phillips, 1958). The theory, developed after this finding spoke of how during times of high demand, firms employ more workers, leading to a tighter labour market. This puts upward pressure on wage claims and, hence, on firm operating costs, which are reflected in higher prices for goods and services. Low demand generates the opposite effect. Thus, favourable demand-side shocks boost economic activity, lowering unemployment; subsequently, we should observe an increase in inflation.4

In the past policymakers believed they could exploit this trade-off, and reduce unemployment at the cost of faster growth in prices. However, advances in the theory behind the Phillips curve, in particular the incorporation of people’s expectations in the late 1960s, as well as a better framework for

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2 Costâncio, V. (2015)

3 In a controversial paper Gordon (2013) shows how his ‘Triangle model’ is able to track inflation well, dismissing claims about the ‘missing deflation’.

4 Favourable supply-side shocks, such as lower oil prices, on the other hand tend to boost economic activity and lower inflation. For instance, it is shown in Grech and Micallef (2014) that a 20% drop in oil prices lowers inflation by 0.74 percentage point after three years in Malta. Conversely, a rise in government expenditure of 1% of GDP raises inflation by 0.47 percentage point over the same period.
firms’ pricing behaviour in the 1970s and 1980s, showed that exploiting this trade-off did not really pay off in the medium to long run.\(^5\) Today the Phillips curve is a complex but important component of the New Keynesian micro-founded models, which are the workhorse models in academia, central banks and other policy making institutions. Despite the rich theory behind it, recent studies have shown that simple versions of the Phillips curve can nonetheless summarise developments in inflation reasonably well. A more detailed review of the literature is given in section 2.

Not much work has been carried out on understanding the Phillips curve in Malta, however the relationship is embedded within the Central Bank of Malta’s macro-econometric model (Grech and Micallef, 2014), whereby inflation responds to economic activity in the short run. In a study documenting changes to the Maltese economy since Independence, Grech (2015a) plots annual data for the unemployment rate and inflation over the period 1960-2014 and finds a negative relationship, in line with theory. The author argues however that the link is weak, implying that a tight labour market does not lead to strong upward pressure on prices, and vice versa.

This paper derives the necessary estimates of economic slack in section 3 and presents a more in-depth analysis of the Phillips curve in the Maltese economy in section 4. It is one of few studies which use data for the Maltese economy dating back to 1965. Various specifications which allow for parameter changes are able to show that the link between economic activity and inflation was high in the mid-1980s but fell progressively over the 1990s and was very weak thereafter. Further analysis expands on this finding by showing that the Phillips curve is subject to threshold effects. The relationship is asymmetric as the link exists mainly during periods of economic revival, but not during an economic slowdown. In addition economic slack must be significant in order to exert pressure on inflation. Estimates of the Wage Phillips curve hint at an opposite behaviour; wage growth is affected mainly during recessions, but again this channel operates when economic slack is sizeable.

The implications of these findings are discussed in section 5, whereby it is argued that increased globalization and lower barriers to trade, mainly through EU accession, led to an increase in competition, putting a lid on price pressures. The results of this paper confirm that import price shocks have played an increasingly important role in affecting inflation in Malta over time. In addition, increased participation in the labour market, especially by females together with an increased inflow of foreign workers boosted the labour supply. Furthermore estimates of the Wage Phillips curve suggest roughly full pass-through of productivity improvements to wages since the 1980s.

2 The Phillips curve

The specification of the Phillips curve has been rigorously developed over time, particularly with the incorporation of inflation expectations in the late 1960s as well as micro-founded derivations of profit maximisation subject to nominal rigidities in the 1970s and 1980s.\(^6\) The hybrid version of the New Keynesian Phillips curve (Galí and Gertler, 1999; Galí et al., 2001; Galí, 2008) is specified as:

\[
\pi_t = \gamma_f \pi_{t+1} + \gamma_b \pi_{t-1} + \lambda \hat{mc}_t
\]

\(^5\)This is because economists realised that as people come to expect higher inflation, say due to increased government spending to boost activity, unions would call for higher wage growth, which would increase unemployment back to the ‘equilibrium’ level. When this point is reached, there would be no more upward pressure on price and wage growth, so the economy would return to the previous unemployment rate, yet it will have a higher rate of price inflation. Thus, the Phillips curve is vertical in the long run.

\(^6\)See Kajuth (2012) for a list of the important contributions to this area.
whereby the parameters $\gamma_f, \gamma_b$ and $\lambda$ are functions of structural parameters, $\pi_{t+1}^e$ is future expected inflation, reflecting forward-looking behaviour, $\pi_{t-1}$ is lagged inflation, capturing inflation inertia, and $\widehat{mc}_t$ is real marginal cost of production, which is the activity variable through which prices are affected. The latter term has been shown to be proportional to the output gap under a number of assumptions (Gali and Gertler, 1999; Gertler and Leahy, 2008). For this reason empirical studies proxy real marginal costs by a measure of the output gap (see Bermingham et al. (2012) and Jordan and Vilm (2014)).

Other studies use the deviation of the unemployment rate from the Non-Accelerating Inflation Rate of Unemployment (NAIRU) as the activity variable, referred to as cyclical unemployment or the unemployment gap (see Ball and Mazumder (2011); Peach et al. (2011); Bermingham et al. (2012); Kajuth (2012); Simon et al. (2013); ECB (2014) and Speigner (2014)). Using cyclical unemployment as the activity variable is more reminiscent of the traditional Phillips curve. Other studies used more complex specifications which take into account asymmetric/threshold effects and differences between the short term and long term unemployment; see inter alia Laxton et al. (1999), Bermingham et al. (2012), Speigner (2014) and Ball and Mazumder (2015).

The definition of expected inflation varies across empirical studies. While in some expectations are forward-looking and are proxied by survey-based measures of expected inflation (Jordan and Vilm, 2014) or announced central bank targets (Simon et al., 2013), in others expectations are backward looking and represented by moving or long run average inflation rates (Ball and Mazumder, 2011).

It is customary in empirical studies to estimate a reduced-form Phillips curve, whereby the aim is to determine the size and significance of the coefficients relating to the determinants of inflation. For instance Simon et al. (2013) and Blanchard et al. (2015) list the following specification:

$$\pi_t = (1 - \vartheta)\pi_{t-1} + \vartheta \pi_t^e - \kappa \bar{u}_t + \gamma \pi_t^m + \epsilon_t$$

whereby $\vartheta$ measures the relative importance given to expectations of future inflation during wage and price setting, relative to information from past inflation, $\kappa$ measures the slope of the Phillips curve on the activity variable (in this case cyclical unemployment $\bar{u}_t$) and $\gamma$ measures the impact of imported inflation.

Recently authors have introduced time variation in the parameters, allowing the relationship between inflation and its determinants to change over time (Simon et al., 2013; Stevens, 2013; Álvarez and Urtasun, 2013; Oinonen and Paloviita, 2014; Riggi and Venditti, 2015). This was partly motivated by the poor forecasting performance for inflation during and after the financial crisis (ECB, 2015). While one reason behind the large forecast errors was incorrect real-time estimates of activity gaps, it has also been shown that the sensitivity of inflation to activity has changed recently. This highlights the importance of allowing for structural change in empirical models.

Furthermore other authors have argued that the Phillips curve may also be subject to threshold and asymmetry effects (Laxton et al., 1999; Musso et al., 2009; Bermingham et al., 2012; Speigner, 2014). The idea behind such specifications is that large, ‘unusual’ deviations in the activity variable may

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7 However it has been argued that for such specifications the proper proxy for marginal costs is the labour share of income; see Galí and Gertler (1999).
affect inflation by a different magnitude than small ‘typical’ deviations; and periods of excess demand and deficient demand of the same magnitude may have a different effect on inflation in absolute terms. These issues are explored in section 4.3.

3 Data

This section describes the data that were used in this study. Part of the data can be found in the statistical appendix in Grech (2015a). This study uses data on the Retail Price Index (RPI), registered unemployment, GDP, foreign consumer prices and wages (compensation per employee). Further information on the data sources and workings can be found in Appendix A.

Figure 1 shows yearly growth in the Retail Price Index (RPI) and an index of foreign consumer prices since the 1960s. The long-run co-movement between these two price series has been high, which implies that both series were driven by common factors, such as the oil price shocks in the 1970s and 1980s. The two series deviate somewhat in the mid-1980s, partly on account of the price controls that were enacted on some consumer prices at the time in Malta. Subsequently, both foreign and domestic inflation stabilised and fluctuated around an average of about 1.9% and 2.3% respectively between 1995 and 2014. Starting in 2007 foreign and domestic consumer prices moved very much in tandem as they were affected by the same energy and food price shocks.

Figure 1: Inflation indicators

Figures 2 and 3 show GDP growth and the unemployment rate and their underlying trend and cyclical components. The latter were estimated from an Unobserved Components Model (UCM) using the Kalman filter; see Appendix B for details. The data match the stylised facts for a number of advanced economies, in which GDP growth became more stable after the 1980s (Summers, 2005) and the Maltese economy, being small and very open, enjoyed the spillovers of the so-called Great Moderation. The output gap and cyclical unemployment are treated as indicators of excess demand; a positive output gap implies that output was growing at a faster rate than potential growth, and vice-versa. Similarly negative cyclical unemployment indicates an excess demand for labour, and vice-versa.9

8This index is based on consumer price index (CPI) developments in France, Germany, Italy and the United Kingdom, which were historically the most important trading partners. See Appendix A for details.

9The output gap and unemployment gap as expected are negatively correlated; and between 1970 and 2014 the highest correlation occurs at a lag of 2-3 quarters at around -0.82. This implies that developments in GDP growth typically precede developments in the labour market with a lag of about 2 to 3 quarters.
The magnitude of the drop in economic activity following the financial crisis in 2008-9 was large by historical standards when measured by the output gap, and is matched only by the recession of the early 1970s. However the 2009 slowdown caused a more modest increase in unemployment, particularly because the decline in economic activity was short-lived. The extent to which these gap variables help explain developments in inflation is explored next.

Figure 2: Real GDP growth and Potential growth (%)

Figure 3: Registered unemployment rate and the NAIRU (%)

4 Estimating a Phillips curve for Malta

4.1 A standard Phillips curve

A general specification for a simple version of the Phillips curve is given by:

$$\pi_t = \alpha X_{t-i} + \gamma \pi^M_{t-i} + \rho \pi_{t-i} + c + \varepsilon_t$$  \hspace{1cm} (3)

whereby $\pi$ is yearly RPI inflation, $X$ is the activity variable and $\pi^M$ is relative import prices proxied by year-on-year growth in foreign CPI less RPI growth. The appropriate lag length $i$ for each variable is determined empirically. Note that this specification assumes that inflation expectations are purely backward-looking, or adaptive. Equation (3) was estimated using OLS using both cyclical

10One must interpret these numbers with care as output and unemployment gap estimates are unobserved variables and therefore are both subject to a degree of uncertainty and may also be sensitive to different modelling assumptions.

11Expressed in terms of deviations from its long run average. In the estimation of the model this variable was lagged by 1 period to avoid endogeneity bias.
unemployment $\bar{u}$ and the output gap $\bar{y}$ as the activity variables ($X \in \{\bar{u}, \bar{y}\}$).

Since $\bar{u}$, $\bar{y}$ and $M$ are all autocorrelated over time, including several lags of these variables would have introduced high multicollinearity between regressors. Hence after some testing cyclical unemployment was only included in its third lag and imported inflation included only in its first lag. This choice was guided by the cross-correlogram for the dependant variable and the regressors, and the lag at which there was the highest correlation was chosen. Given the co-movement of the output gap and cyclical unemployment, in the output gap version of the model the output gap was lagged by 1 quarter. Since inflation is measured in annual percentage changes, the model includes both the first and the fourth lag of inflation to control for residual serial correlation. Furthermore inference is based on Newey-West standard errors.

The results of these OLS estimations are shown in Table 1 below. The ‘slope’ of the Phillips curve, $\alpha$, is statistically significant at conventional levels only when cyclical unemployment is used as the activity variable. This shows some link between economic activity and prices. It can be argued that cyclical unemployment is a more indicative measure of economic activity than the output gap, as transitory shocks to GDP, which affect the output gap, may be absorbed by firms and thus not reflected in a change in unemployment. Column (1) also confirms the important role of import price shocks on domestic inflation. The measure of fit of both models is very high, although most of the fit can be attributed to inflation being explained by its own history.

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<tr>
<td>$\bar{u}_{t-3}$</td>
<td>-0.482 **</td>
<td>-0.584 **</td>
<td>-0.284</td>
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<td>$\bar{y}_{t-1}$</td>
<td>0.009</td>
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<td>$\pi_{M_{t-1}}$</td>
<td>0.114 **</td>
<td>0.109</td>
<td>0.134 **</td>
<td>0.295 **</td>
<td>0.123</td>
<td>0.338 **</td>
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<tr>
<td>$\pi_{t-1}$</td>
<td>1.050 ***</td>
<td>1.069 ***</td>
<td>1.086 ***</td>
<td>0.908 ***</td>
<td>1.098 ***</td>
<td>0.975 ***</td>
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<tr>
<td>$\pi_{t-4}$</td>
<td>-0.207 ***</td>
<td>-0.203 ***</td>
<td>-0.215 ***</td>
<td>-0.342 ***</td>
<td>-0.210 ***</td>
<td>-0.329 ***</td>
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<tr>
<td>$c$</td>
<td>0.296 **</td>
<td>0.256</td>
<td>0.096</td>
<td>1.113 **</td>
<td>0.088</td>
<td>0.939 **</td>
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<td>$\bar{R}^2$</td>
<td>0.857</td>
<td>0.851</td>
<td>0.867</td>
<td>0.638</td>
<td>0.859</td>
<td>0.634</td>
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<td>S.E.</td>
<td>1.1949</td>
<td>1.2177</td>
<td>1.4089</td>
<td>0.6745</td>
<td>1.4462</td>
<td>0.6785</td>
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Note: *, ** and *** imply statistically significant results at the 5% and 1% level of significance respectively and are based on Newey-West standard errors.

These estimates are based on a relatively long time series during which the Maltese economy witnessed significant structural and socio-economic changes. Consequently it is likely that the relationship presented above might have changed over time. Two approaches were used to test the stability of the parameters. In the first the sample was split into two, an ‘early’ period spanning 1966-1995, and a more recent period between 1996 and 2014, and both versions of the Phillips curve were estimated within each sub-sample. The results can be found in columns (3) to (4) and (5) to (6) respectively in

12 See Grech (2015a).
Table 1. This approach tracks changes in the parameters over the two periods.

The second approach involved rolling regression estimates of equation (3) to track the evolution, if any, of the parameters over time. Starting from 1966Q4, the first 80 observations (the ‘window’ - in this case 20 years) were used to estimate the Phillips curve. The resulting parameters were saved, and the sample was moved by one period forward in time, while keeping the same window length, and the process was repeated until the end of 2014. The estimates from each recursion track ‘smooth’ changes in the parameters as time series, which are shown in Figure 4.

Both the sub-sample and rolling regression approaches provide evidence of a change in the slope of the unemployment gap version of the Phillips curve $\alpha$ over time. The parameter in the first sub-sample is -0.584 and statistically significant, while in the second it is lower in absolute terms (-0.284) and not statistically different from zero, indicating a significant change. This development is not easily seen in the output gap version, as against expectations both sample periods yield a negative and insignificant slope.

Meanwhile in the left panel of Figure 4 we see that the slope of the Phillips curve was relatively stable for some time, although estimated with high uncertainty. It increased in absolute terms for a short while during the 1990s but subsequently fell towards 0. Similar dynamics can also be seen in the economy’s sensitivity to import price shocks, despite the wide confidence interval. Towards the end of the sample the effect of import price shocks is observed to have risen steadily.

These results are interesting as they confirm significant structural changes in the Maltese economy took place. Hence the estimates presented in Table 1 represent ‘average’ values of the coefficients across time. Furthermore and as noted above, volatility in inflation was high in the 1970s and 1980s but then fell markedly since the 1990s. This phenomenon is also observed in many major economies as one of the characteristics of the Great Moderation. The fall in inflation and inflation volatility across advanced economies is, inter alia, a consequence central banks gaining credibility as inflation targeting

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13 This approach is frequently used in the literature, see Oinonen and Paloviita (2014).
14 The first set of parameter estimates that are available are dated 1985Q3. The results using the output gap version of the model are not shown, but are qualitatively the same.
15 The timing of these dynamics are somewhat sensitive to the window length, as a shorter window of say 60 observations shows the same dynamics occurring earlier in time. The key implication is the indication of instability in the relationship over time.
institutions. This led to better management of and hence stability in long-term inflation expectations (Simon et al., 2013; Carney, 2015). Yet the results presented above rely on the assumption that the variance of inflation was constant over the entire sample period, which is a strong assumption not supported by the data.

4.2 Incorporating time variation: a TVP-SV model

These results motivate the use of a more flexible model, which is able to capture the changes hinted at above in the relationship between inflation and its determinants in real-time. Moreover, a model which allows the parameters to change over time but ignores the changing volatility in the dependant variable is likely to overestimate or lead to spurious variation in the coefficients, as these ‘soak up’ some of the variance of the residuals (Cogley and Sargent, 2005; Primiceri, 2005; Nakajima, 2011). For this reason a Time-Varying Parameter model with Stochastic Volatility (TVP-SV) is explored next. This model jointly allows the parameters and volatility of shocks to inflation to change over time. What follows focuses on the unemployment gap version of the model.

The TVP-SV specification of the baseline Phillips curve is the following

\[ \pi_t = \alpha_t \bar{u}_{t-3} + \gamma_t \pi^M_t + \rho_{1,t} \pi_{t-1} + \rho_{4,t} \pi_{t-4} + \epsilon_t + \sqrt{h_t} \]  

(4)

whereby shocks to inflation \( \epsilon_t \) are augmented with a time-varying variance term \( h_t \). The parameters of the model \( \alpha, \gamma, \rho \) and \( c \) and the logarithm of \( h \) are assumed to follow random walks. If these are grouped in the vector \( B_t = [\alpha_t \ \gamma_t \ \rho_{1,t} \ \rho_{4,t} \ c_t] \prime \), then the evolution of the parameters can be represented as:

\[ B_t = B_{t-1} + \nu_t \]  

(5)

whereby \( \nu_t \) is a diagonal matrix of shocks. The evolution of the (log) variance of shocks is given by:

\[ \log h_t = \log h_{t-1} + \eta_t \]  

(6)

where \( \eta \) is a disturbance term. This setup constitutes a non-linear state-space model, as the state variable \( h_t \) is not linear in the observation equation (eq. 4). The model is estimated using Bayesian methods, specifically a Metropolis-within-Gibbs sampler, using the algorithm of Carter and Kohn (1994) to extract the path for all the elements in \( B_t \) in every iteration. Following Primiceri (2005) a fraction of the data were used in a training sample to initialize the priors (1966Q4–1979Q4). More details are available in Appendix C. The sample on which inference is based spans 35 years (1980Q1–2014Q4). The results, which are based on the posterior distributions of the parameters, are shown in Figure 5. The variance of shocks to inflation \( h_t \) (bottom right panel) exhibited significant time variation, being high in the early 1980s but then falling significantly. Inflation volatility rose temporarily just before the 1990s and more recently in 2007, on account of the food price shocks that preceded the financial crises. This confirms that the assumption of constant variance in the model presented in the previous section is too restrictive.

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16 Carney (2015) argues that this can be seen by comparing the correlation of global headline and core inflation over time; which was high in the 1970s but has fallen since then.

17 See Cogley and Sargent (2005) and Primiceri (2005) and the literature cited therein for a discussion of this model.

18 See Jacquier et al. (1994) and Blake and Mumtaz (2012) for a discussion of Monte Carlo Markov Chain (MCMC) Bayesian inference in such models.
The time-varying Phillips curve slope $\alpha_t$ is estimated to have declined in absolute terms since the 1980s; implying a weakening in the relationship between economic activity and inflation. This confirms the findings of the previous section; it is a pattern that was observed in many advanced economies, in line with the findings in Simon et al. (2013) and more recently Blanchard et al. (2015). Although there are no major changes during the period associated with the financial crisis, a slight but sudden change in the trend of the slope can be seen starting in 2010.

The estimates for $\gamma_t$ show that the role of import prices was weakest during the period of the price controls in the early 1980s. Thereafter imported price shocks played a progressively stronger role in explaining inflation in Malta. This trend is also in line with other studies Stevens (2013) and Simon et al. (2013), who find the same behaviour in the economies of the EU and of a number of OECD countries respectively.

Figure 5: Phillips curve: TVP-SV estimation results

Inflation persistence fell significantly since the 1980s; implying that, keeping everything else constant, shocks to inflation used to die off much slower in the past, and as a result the cumulative impact was bigger. The anchoring of expectations in many major economies and the increased synchronisation of the Maltese economy with such developments is perhaps the key reason for the local decline in inflation persistence. The uncertainty surrounding these parameter estimates is quite high, as the 68% credible intervals are relatively wide for $\alpha$, $\gamma$ and $\rho$. Nevertheless these results highlight the possible changes that have occurred in the macroeconomy, and are discussed further in section 5.
The estimates presented above could be sensitive to how economic slack is measured. To this end the model was re-estimated using a measure of cyclical unemployment derived from the Hodrick-Prescott filter, which is another frequently used trend-cycle decomposition technique. The flattening of the Phillips curve is also observed on the basis of this activity measure.\textsuperscript{19} Interestingly the sudden reversal in the slope starting in around 2010 (see Figure 5) is also confirmed in this set of estimates.

Furthermore other variables were identified as proxies for imported inflation: the import price deflator and a measure of competitors’ import prices used in Eurosystem projection exercises; see Hubrich and Karlsson (2010, p.13) for details.\textsuperscript{20} These two indicators are available for the periods 1955Q1-2014Q4 and 1980Q1-2014Q4 respectively, and are discussed further in Appendix B. The TVP-SV model presented above was re-estimated using these two proxies. The same flattening of the Phillips curve is observed, and there is some further evidence of an increase in $\gamma$ over time, although not to the extent as in the baseline results. This could be due to the fact that these two indices track all prices, not just consumer prices, and hence are poor proxies for import price shocks to consumer prices.

### 4.3 The role of asymmetry and threshold effects

The model specified above postulates that the economy behaves the same way irrespective of the state it is in, that is, irrespective of whether it is going through a boom or recession. This goes against the literature on downward nominal rigidities and Phillips curve convexity, which argues that one should not expect the same relationship at different points along the Phillips curve.\textsuperscript{21} There is also empirical evidence that the response of inflation to slack may be state-dependent. For example Demers (2003) and Barnes and Olivei (2003) find that the slope of the Phillips curve can change according to the state of the economy within a business cycle. Berningham et al. (2012) also find the existence of threshold effects, whereby the link between economy and inflation is stronger during recessions compared to booms. Further discussion can be found in Musso et al. (2009).

To allow for the possibility of asymmetries in the Phillips curve, the model in the previous section was modified to the following specification:

$$
\pi_t = \alpha_t \tilde{u}_{t-3} \cdot I(\tilde{u} < 0) + \alpha_t \tilde{u}_{t-3} \cdot I(\tilde{u} > 0) + \gamma_t \pi^M_t + \rho_{1,t} \pi_{t-1} + \rho_{4,t} \pi_{t-4} + c_t + \varepsilon_t \sqrt{h_t}
$$

whereby $I(.)$ is the indicator function which takes a value of 1 when the condition within the brackets holds. This threshold model relaxes the assumption of a symmetric Phillips curve and allows for distinct slopes during different stages of the economic cycle; namely different slopes during recessions ($\tilde{u} > 0$) and booms ($\tilde{u} < 0$). As above, the parameter vector $B_t$ and the (log) variance $\log(h_t)$ evolve as random walks. Estimation follows the same procedure as above, and the results are summarized in Figure 6.

There is evidence of asymmetry in the Maltese Phillips curve. The slope associated with recessions has fallen in absolute terms and turned positive, while that associated with expansions has not changed much since the 1980s. Thus the threshold model decomposes the fall in the baseline Phillips curve slope discussed in the previous section; the flattening phenomenon appears to be driven mainly by

\textsuperscript{19}Robustness estimation results are not shown but are available from the author on request.

\textsuperscript{20}This index is similar in methodology to the index of foreign consumer prices used in the baseline results, although it is based on total export prices of other countries, not only prices for household tradable goods.

\textsuperscript{21}See Laxton et al. (1999), and more recently Speigner (2014). A theoretical account of how rigidities affect the convexity of the Phillips curve is given by Daly and Hobijn (2014).
behaviour during economic slowdowns, rather than a general fall in the sensitivity of inflation to economic conditions.

Figure 6: Asymmetric Phillips curve: TVP-SV estimation results

It was noted that the estimated unemployment gaps varied in magnitude over time; large deviations from the NAIRU occurred mostly during the early part of the sample. It may be the case that the Phillips curve relationship may also be sensitive to the size of labour market slack, as discussed in Barnes and Olivei (2003). The implication of this argument would be that the observed flattening may not reflect a change in the relationship, but merely the fact that prices are nowadays reacting to much smaller shocks than in the past.

To test this formally, while maintaining the separation between periods of booms and recessions, the Phillips curve was modified to the following form:

\[ \pi_t = \left[ \alpha_t \tilde{u}_{t-3} \cdot I(\tilde{u} < 0) + \alpha_t \tilde{u}_{t-3} \cdot I(\tilde{u} > 0) \right] \cdot I(|\tilde{u}| < \kappa) \\
\left[ \alpha_t \tilde{u}_{t-3} \cdot I(\tilde{u} < 0) + \alpha_t \tilde{u}_{t-3} \cdot I(\tilde{u} > 0) \right] \cdot I(|\tilde{u}| > \kappa) + \gamma_t \pi_t^M + \rho_{1,t} \pi_{t-1} + \rho_{4,t} \pi_{t-4} + c_t + \varepsilon_t \sqrt{h_t} \]  

(8)

where the indicator function outside square brackets switches on during periods of low (|\tilde{u}| < \kappa) or high (|\tilde{u}| > \kappa) labour market slack respectively. This specification nests that in equation 7, so that
the threshold effect is tested along with the asymmetry effect discussed above. The threshold value \( \kappa \) was set at 1 standard deviation of the unemployment gap.\(^{22}\) This specification effectively allows the economy to be in four distinct states and returns four slope parameters, which describe the relationship between inflation and slack during shallow and deep recessions and small and large booms.

The results, shown in Figure 7, shed further light into the degree of asymmetry and sensitivity of inflation to small and large shocks. The estimates are subject to a higher degree of uncertainty, given that fewer observations are available in each state, so these results should be interpreted with some caution.

Nevertheless the results related to the asymmetry of the Phillips curve discussed above remain valid. Shallow recessions have not been associated with a drop in inflation, and although there is evidence that deep recessions may have put downward pressure on inflation in the 1980s, the link has since then disappeared. Similarly small expansions have not been associated with a rise in inflation. The asymmetry takes effect only during large expansions, and these estimates suggest that this relationship has not changed by much since the 1980s.

Figure 7: Asymmetric Threshold Phillips curve: TVP-SV estimation results

\( ^{22}\)Similar results were obtained at a lower (0.5 standard deviation) and higher (2 standard deviations) threshold.
4.4 The Wage Phillips curve

The analysis can also be extended to the Wage Phillips curve, which tracks the influence of economic activity on wage growth. A similar specification which tests for the presence of asymmetry in the relationship is given by:

\[
\pi_t^W = \alpha_t \tilde{u}_t \cdot I(\tilde{u} < 0) + \alpha_t \tilde{u}_t \cdot I(\tilde{u} > 0) + \theta_t \Pi_t + \rho_t \pi_{t-1}^W + c_t + \varepsilon_t \sqrt{h_t} \tag{9}
\]

whereby \(\pi^W\) and \(\Pi\) are wage and productivity growth respectively, and, as above, \(\tilde{u}\) is cyclical unemployment. Such a specification nests a symmetric response. This equation was estimated using annual data from 1982 to 2014, and the results are shown in Figure 8.\(^{23}\) Similar to the price Phillips curve, a threshold model separating the economy in periods of low and high divergences from equilibrium was also specified in equation 10.

\[
\pi_t^W = \alpha_t \tilde{u}_t \cdot I(|\tilde{u}| < \kappa) + \alpha_t \tilde{u}_t \cdot I(|\tilde{u}| > \kappa) + \theta_t \Pi_t + \rho_t \pi_{t-1}^W + c_t + \varepsilon_t \sqrt{h_t} \tag{10}
\]

Given that annual data is used in this estimation, the low number of observations does not allow the separation of the data in 4 states as for the price Phillips curve. The results of this threshold model are shown in Figure 9.

Both sets of the Wage Phillips curve estimates show a link between economic activity and wage growth (see Figures 8 and 9). The asymmetric model reveals that this channel operates mainly through slower wage growth during recessions. On the other hand periods of strong economic growth were not associated with faster wage growth \textit{ceteris paribus}. The effect is estimated to have declined slightly since the mid-1980s, however it still explains the existence of a link between economic activity and wage growth in the Maltese economy. Furthermore the threshold model reveals that wage growth responds mainly to large deviations in unemployment from the NAIRU. Combining these two results together, it appears that wage growth is affected mainly during big recessions.

The parameter \(\theta\) shows the contribution of productivity growth to wage inflation. The long run contribution (given by \(\frac{\theta}{1 - \rho}\)) is just under 1 over the whole sample, implying that, keeping everything else constant, productivity improvements were passed on to higher wages throughout the period 1980-2014.\(^{24}\) Furthermore the results above show that, similar to price growth, shocks to wage growth became less volatile, mainly from the year 2000 onwards.

5 What caused a flat Phillips curve?

A number of theories have been fielded to explain changes in the Phillips curve slope. The central argument raised in studies which report a flattening of the Phillips curve is a general move towards ‘anchored inflation expectations’ (Simon et al., 2013; Ball and Mazumder, 2015; Blanchard et al., 2015). People’s belief of moderate and stable future inflation, brought about by successful central bank monetary policy, reduced pressure on wages by workers and unions seeking to maintain the purchasing power of income. Anchored inflation expectations were attributed to the so-called ‘missing deflation’ in OECD countries, whereby economic activity dropped significantly but inflation did not turn negative.

Another key argument is the role of globalisation. Lower global inflation, in part due to increased

\(^{23}\)Data for compensation was not available on a quarterly basis and being a dependant variable, it was decided not to interpolate it to quarterly frequency. The set-up and estimation method are otherwise the same as discussed above.

\(^{24}\)It can be noted that when persistence rose, the parameter \(\theta\) fell, maintaining the ratio close to 1.
openness to trade and cheaper imported goods - the so-called “China effect” (Lewis and Saleheen, 2014) - lowered domestic inflation. To this end changes over both the general level of mark-ups and their relation to the economic cycle might have also changed pricing behaviour, and hence affected the Phillips curve slope (Carney, 2015). A theoretical account of how globalization drives a flat aggregate supply curve is given in Razin and Binyamini (2007). In fact, the empirical work of Borio and Filardo (2007) presents cross-country evidence of an increased role for global factors in explaining domestic price developments, especially since the 1990s. Furthermore Sbordone (2007) argues that globalisation may have led to a low inflation environment by moderating growth in marginal costs through increased competition.

Using a New Keynesian Phillips curve as specified in equation (1) above, Kuttner and Robinson (2010) argue that changes in the persistence of marginal cost fluctuations can lead to a flattening of the slope, which is typically observed in reduced form estimates. However they show that their estimate of the structural parameter $\lambda$ linking developments in marginal costs and inflation in the United States fell over time through an increase in the Calvo parameter - the probability that firms in any point in time cannot revise prices - such that the observed flattening is not simply a reduced-form phenomenon.

Since the Maltese economy is very small and open, the globalization argument is considered the prime mechanism driving a flatter Phillips curve. Lower barriers to trade over time, brought about by EU accession and later the adoption of the euro, led to increased competition, which controlled price pressures. This was coupled with low and stable inflation in trading partner countries. A more recent
phenomenon, the rise of online purchases from abroad, marks an additional development in product market competition. In fact whereas only 34% of Maltese households with internet access had purchased goods online in 2005, this percentage rose to 66% by 2015. All of these developments have led to a decline in trend inflation in Malta (Gatt, 2014).

While estimates of the asymmetric Phillips curve showed that the flat slope relates only to periods of subdued economic activity, the impact during booms is modest. Based on the median estimate of the slope at the end of the sample, a 1 standard deviation downward shock to the unemployment gap increases inflation by about 0.23 percentage point on impact, falling to 0.12 percentage point after 5 quarters. Turning to the Wage Phillips curve, there is also evidence of asymmetric behavior. The median estimate of the slope at the end of the sample suggests that a 1 standard deviation upward shock to unemployment relative to the NAIRU causes wages to fall by about 0.9 percentage point within the same year.

Developments in the labour market may have contributed to lower pressure on wage growth. Trade unionisation rates have declined significantly from 33% in 1995 to 23% in 2013 (Micallef and Caruana, 2014). Labour participation rates, which were stable for decades, rose sharply after 1995, led by a near doubling of the female participation rate. This was also complemented by a significant inflow of foreign workers following EU accession (Grech, 2015b), and hence an overall increase in the labour supply may have dampened wage claims. These developments may be behind the stabilization in trend wage inflation, and thus explain the fall in the volatility of wage growth as shown in Figures 8
and 9. Therefore while in Malta inflation tends to rise during an expansion, wage growth in general
does not. Conversely, during an economic slowdown inflation does not fall but wage growth falls if
slack is high.

6 Conclusion

This paper discusses and presents estimates of the Phillips curve in the Maltese economy using data
starting from the mid-1960s. While OLS regression results show that the data fit the relationship
over the full sample, sub-sample estimates point to a weakening of the relationship over time. Mean-
while the same analysis shows an increase in the sensitivity of domestic inflation to import price shocks.

To analyse this further a more flexible model which allows the Phillips curve parameters to change
over time was employed. Estimation of this model was based on a mix of Bayesian methods, and the
results show significant changes in the parameters over time. The model is also able to track changes
to the variance of shocks affecting inflation, and the results show that recently shocks became smaller
on average compared the 1980s, peaking only during the energy and food price shocks of 2007. A
decomposition of the sources of inflation shocks shows that import price shocks were the main factor
affecting inflation since the late 1990s.

The decline in the slope of the Phillips curve is shown to be due to an asymmetry in the relation-
ship; the link between economic activity and inflation exists only during times of growth, implying
downward price rigidity. Opposite results hold for the Wage Phillips curve, the link between economic
activity and wage growth holds only during slowdowns. In both cases the link between activity and
nominal variables was found to exist only when the shock to the economy is sizeable.

Increased openness is the key driver for the observed flattening of the price Phillips curve. The boost
in the supply of labour through increased participation as well as an inflow of workers from abroad
has kept upward pressure on wage inflation in check during growth phases. However the estimates
suggest that productivity improvements have generally been rewarded in full since the 1980s.
References


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Appendix A  Data

Inflation

Inflation is measured using the Retail Price Index, an index which was first estimated in 1936 (Camilleri, 1996) and has been the key indicator used to track inflation in Malta for many decades. This data was obtained in monthly frequency for the period 1950-2014 from the National Statistics Office. Since the index was re-based several times since 1950, the index was spliced into a consistent series and re-based.\(^{25}\) Quarterly averages of the price index \(P_t\) were obtained and inflation is defined as

\[
\pi_t = \left[ \log(P_t) - \log(P_{t-4}) \right] \times 100.\]

Unemployment rate

Data for registered unemployed and the labour supply were obtained from the Employment and Training Corporation (ETC) from 1965 to 2014 in monthly frequency and spliced into a consistent series. The series was then seasonally adjusted using Census X12.

GDP

Annual data for GDP was obtained as a spliced series from different statistical methodologies over time.\(^{27}\) The whole dataset was re-based in terms of millions of euros based on 2010 prices and was interpolated from annual to quarterly frequency using the Litterman interpolation technique (Litterman, 1983).\(^{28}\),\(^{29}\)

Foreign CPI

This index was constructed as a weighted average using the consumer price indices of four of Malta’s major trading partners, namely the United Kingdom (UK), France, Germany and Italy. These countries accounted on average for more than 80% of all trade within the EU and just under 60% of all of Malta’s trade between the period 1980-2014. CPI indices are those found in OECD (2010)\(^{30}\), and were extracted from the FRED© database. Relative weights were obtained from the historical Direction of Trade statistics that can be found on the Central Bank of Malta’s website. Data prior to 1980 was not available, and thus the earliest datapoint available was held constant to the past.

Import price deflator

The import price deflator was used as a measure of foreign imported prices.\(^{31}\) As for GDP, data from different national accounts frameworks were obtained from the Bank’s website, and the different series

\(^{25}\)It should be noted that the coverage of the RPI basket was initially very limited; but was extended over time to include a wider range of goods and services.

\(^{26}\)Unless otherwise stated all growth rates are calculated using this formula.

\(^{27}\)The data between 1954 and 2000 are those measured by the System of National Accounts (SNA) methodology, and two such series were available, one based on 1973 prices (which spans from 1954-1997) and another based on 1995 prices (data for which spans 1970-1995). GDP data from 2000 onwards is based on the European System of Accounts 2010 (ESA2010) guidelines. All these series were spliced into a consistent and continuous series.

\(^{28}\)Litterman Match Sum technique in Eviews 9

\(^{29}\)Although ESA2010 data is available in quarterly frequency, mixing this data with the interpolated quarterly frequency data would have introduced noise to a relatively smooth data series.

\(^{30}\)OECD (2010), “Main Economic Indicators - complete database”, Main Economic Indicators (database), (Accessed on 31/08/2015)

\(^{31}\)It can be argued that this index also covers prices of industrial goods and goods for re-export, prices which should not have an effect on domestic consumer price inflation.
spanning different periods were spliced into a single time series. This series was interpolated from annual to quarterly frequency using the Litterman interpolation technique. As for domestic inflation, inflation in imported prices $M_t$ was defined as $m = [ln(M_t) − ln(M_{t-4})] \times 100$.

**Competitors’ Import Prices**

This is an indicator of foreign price pressures for the Maltese economy, and forms part of the technical assumptions that are used in the Eurosystem Broad Macroeconomic Projections Exercise (BMPE). For more details see Hubrich and Karlsson (2010). Yearly growth is defined as above.

**Wages and productivity**

Wages are proxied by compensation per employee, which was obtained in annual frequency for the period 1970-2014 from the statistical database in Grech (2015a). Labour productivity was obtained from the same source also in annual frequency. Yearly growth in wages $\pi_W^t$ and productivity $\Pi_t$ was defined as above.
Appendix B  Extracting cyclical indicators

This section describes the Unobserved Components Model (UCM) that was used to extract potential output growth, the output gap, the NAIRU and cyclical unemployment from the GDP and unemployment data. The trend-cycle decomposition was conducted by representing the system as a state space model:

\[
\begin{align*}
\dot{GDP}_t &= \tau_t + \mu_t + \nu_t \\
U_t &= N_t + \lambda_t + \eta_t \\
\tau_t &= \tau_{t-1} + \epsilon^\tau_t \\
N_t &= N_{t-1} + \epsilon^N_t \\
\mu_t &= \rho_1 \mu_{t-1} + \rho_2 \mu_{t-2} + \epsilon^\mu_t \\
\lambda_t &= \rho_1 \lambda_{t-1} + \rho_2 \lambda_{t-2} + \theta \mu_{t-4} + \epsilon^\lambda_t \\
\nu_t &= \epsilon^\nu_t \\
\eta_t &= \epsilon^\eta_t
\end{align*}
\]

whereby: $GDP_t$ - GDP growth, $U_t$ - unemployment rate, $\tau$ - potential output, $\mu$ - output gap, $N$ - NAIRU, $\lambda$ - cyclical unemployment, $\nu / \eta$ - measurement errors, and $\epsilon_i^i$ random shocks to variable $i$. Equations (1) and (2) are the observation equations, which state that the left-hand side variable in each is the sum of a trend, a cyclical component and an irregular component which accounts for measurement errors. These sub-components are the unobserved state variables which the framework tries to identify. Therefore $\tau$ and $N$ represent potential output growth and the NAIRU respectively, and these are modelled in equations (3) and (4) as random walks which are subject to white noise shocks $\epsilon^\tau_t \sim N(0, \Sigma^{\tau})$ and $\epsilon^N_t \sim N(0, \Sigma^N)$.

The output gap and cyclical unemployment are modelled as $\mu$ and $\lambda$ respectively in equations (5) and (6). Owing to their cyclical nature they are modelled as stationary AR(2) processes, however the process generating cyclical unemployment is also a function of the output gap lagged by four quarters, in the spirit of Okun’s law. This later detail adds some economic structure to the decomposition implied by the system. Both of these processes are also subject to random shocks $\epsilon^\mu_t \sim N(0, \Sigma^\mu)$ and $\epsilon^\lambda_t \sim N(0, \Sigma^\lambda)$. The measurement errors $\nu$ and $\eta$ follow white noise processes given by $\epsilon^\nu_t \sim N(0, \Sigma^\nu)$ and $\epsilon^\eta_t \sim N(0, \Sigma^\eta)$. All disturbances are uncorrelated with each other.

The model was parameterised as shown in the table below and run through the Kalman Filter. These parameters were chosen such that the resulting trend variables $\tau$ and $N$ are not excessively volatile but evolve progressively over time. Figure 12 compares the estimate of potential output growth given by this technique to that from a recent study by Grech (2015a), who used the Production Function methodology, and shows that the two measures are highly correlated.
Table 2: Model Parameters

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<th>Coefficients</th>
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</tr>
<tr>
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<td>$\theta$</td>
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</tbody>
</table>

Figure 12: Potential output growth

Note: Potential output from the Production Function was obtained from the study by Grech (2015a) and is the sum of contributions to growth from labour and capital inputs and total factor productivity.
Appendix C  The Time-Varying Parameter model with Stochastic Volatility

The model discussed in section 4.2 is given by

\[ \pi_t = c + \alpha_t \tilde{u}_{t-3} + \gamma_t \pi_{t-1}^M + \rho_{1,t} \pi_{t-1} + \rho_{4,t} \pi_{t-4} + \varepsilon_t \sqrt{h_t} \]  

(1)

\[ \alpha_t = \alpha_{t-1} + \epsilon_t^\alpha \]  

(2)

\[ \gamma_t = \gamma_{t-1} + \epsilon_t^\gamma \]  

(3)

\[ \rho_{1,t} = \rho_{1,t-1} + \epsilon_t^{\rho_1} \]  

(4)

\[ \rho_{4,t} = \rho_{4,t-1} + \epsilon_t^{\rho_4} \]  

(5)

\[ c_t = c_{t-1} + \epsilon_t^c \]  

(6)

\[ \log h_t = \log h_{t-1} + \eta_t \]  

(7)

whereby shocks to inflation \( \varepsilon_t \) are augmented with a time-varying variance term \( h_t \) and the parameters of the model \( \alpha, \gamma, \rho \) and \( c \) and the logarithm of \( h \) are assumed to follow random walks. If the vector \( B \) holds the parameters of the model \( B_t = [\alpha_t \gamma_t \rho_{1,t} \rho_{4,t} c_t]^\prime \), then the evolution of these parameters can be represented as:

\[ B_t = B_{t-1} + \nu_t \]  

(8)

whereby \( \nu_t \) is a diagonal matrix of shocks following the multivariate normal distribution with mean 0 and variance matrix \( Q \):

\[ \nu_t \sim N(0,Q) \]  

(9)

Similarly the stochastic component in the log-volatility transition equation (eq. 6) follows a normal distribution with mean 0 and variance \( g \):

\[ \eta_t \sim N(0,g) \]  

(10)

This setup constitutes a non-linear state-space model, as the state variable \( h_t \) is not linear in the observation equation (eq.7). The model is estimated using Bayesian methods, specifically a Metropolis-within-Gibbs sampler, using the algorithm of Carter and Kohn (1994) to extract the path for all the elements in \( B_t \) in every iteration.\(^{32}\)

Following Primiceri (2005), the prior for \( Q \) follows an inverse Wishart distribution \( (Q \sim IW(Q_0,T_0)) \) with scale matrix \( Q_0 = (Q_{OLS} \times T_0 \times k) \), and \( T_0 \) degrees of freedom, where \( Q_{OLS} \) is the covariance matrix from an Ordinary Least Squares (OLS) regression of the Phillips curve on a training sample, \( T_0 \) is the number of observations in the training sample, and \( k \) is a scaling factor. The training sample spans 1966Q4-1979Q4 \( (T_0 = 53) \), and the value of \( k \) was set to 0.01, which is standard in the literature (see Primiceri (2005); Cogley (2005) and Cogley and Sargent (2005)). A higher \( k \) reflects the prior belief of greater time-variation. Setting \( k^* = 5k \) results in more changes in the parameters within \( B \), while \( k^* = \frac{k}{5} \) produces smoother dynamics, although in both cases the results remain qualitatively similar to those from the baseline settings.

\(^{32}\)See Jacquier et al. (1994) and Blake and Mumtaz (2012) for a discussion of Bayesian inference in such models.
Similarly the prior for \( g \), the variance of shocks to log volatility, follows the inverse Gamma distribution \((g \sim IG(V, S))\), with prior degrees of freedom \( V = 5 \) and scale \( S = 0.5 \). This prior incorporates the belief that volatility shocks to inflation were historically large but places some uncertainty around this belief. As discussed in Section 4.2, allowing for changing error variance reduces the risk of spurious time variation in the parameter vector \( B \).

The estimation of this model proceeds in the following sequence:\(^{35}\)

1. **Sample the process \( h_t \)**

   This procedure is derived in Jacquier et al. (1994) and Jacquier et al. (2004), which involves specifying the distribution for \( h_t \) conditional on \( h_{t-1}, h_{t+1} \) and the data \( Y_t \) as the product of Normal and log-Normal densities:

   \[
   f(h_t|h_{t-1}, h_{t+1}, Y_t) = h_t^{-0.5} \exp\left(-\frac{\epsilon_t^2}{2h_t}\right) \cdot h_t^{-1} \exp\left(-\frac{(\ln h_t - \mu)^2}{2\sigma_h}\right)
   \]

   whereby \( \mu = \ln h_{t+1} + \ln h_{t-1} \) and \( \sigma_h = \frac{g}{2} \). An independence Metropolis-Hastings algorithm was used to draw from the candidate density, which is the second term in (11).

   To sample the initial value of \( h_t \), i.e. \( h_0 \), the authors suggest assuming a prior for \( \ln h_0 \): \( \ln h_0 \sim N(\bar{\mu}, \bar{\sigma}) \) whose posterior density is given by:

   \[
   f(h_0|h_1) = h_0^{-1} \exp\left(-\frac{(\ln h_0 - \mu_0)^2}{2\sigma_0}\right)
   \]

   whereby \( \sigma_0 = \frac{\sigma_0}{\sigma + g} \) and \( \mu_0 = \sigma_0 \left(\frac{\mu}{2} + \frac{\ln h_1}{2}\right) \) which require a value for \( \bar{\sigma}, \bar{\mu}, h_1 \) and \( g \). The hyperparameter \( \bar{\mu} \) is estimated as the log of the variance of the OLS regression residuals, while \( \bar{\sigma} \) is set to a high number to reflect the uncertainty around this estimate. In practice values for \( \bar{\sigma} \) between 10 and 200 do not affect the results in a meaningful way. An estimate of the process \( h_t \) was obtained as the sequence of squared changes in the dependent variable in equation (1) (inflation), and the value for \( h_1 \) is simply the first number in this series. The value of \( g \), the variance of the process \( \ln h_t \), was initialised to 1.

   The process to sample the sequence \( h_{t=1} \) to \( h_{T-1} \) (conditional on \( g \) and \( B_t \) ) involves sampling from the density in (12) with \( h_0 = h_t, \mu = \ln h_{t+1} + \ln h_{t-1} \) and \( \sigma_h = \frac{g}{2} \). This draw is retained with probability

   \[
   \alpha = \min\left(\frac{h_{t,\text{new}}^{-0.5} \exp\left(-\frac{\epsilon_t^2}{2h_{t,\text{new}}}\right)}{h_{t,\text{old}}^{-0.5} \exp\left(-\frac{\epsilon_t^2}{2h_{t,\text{old}}}\right)}, 1\right) > u \quad u \sim U(0, 1)
   \]

   i.e. if \( \alpha \) is greater than a draw between 0 and 1 from the uniform distribution, the new draw \( h_{t,\text{new}} \) is accepted, otherwise the previous draw is retained.

   Finally, the value for \( h_T \) is sampled from the same density in (12) with \( \mu = \ln h_{t-1} \) and \( \sigma_h = g \) and the same acceptance probability is calculated.

2. **Sampling \( g \)**

   For each full sequence \( h_t \) constructed above, the residuals from the transition equation (6) (\( \eta_t \))
were calculated and a value for $g$ was drawn from the inverse Gamma distribution with degrees of freedom $T + V$ and scale $\Sigma \eta_t^2 + S$.

3. **Extracting $B_t$**

Conditional on $h_t$ and $Q$, the processes for the time varying parameters were drawn using the Carter-Kohn algorithm (Carter and Kohn, 1994).

4. **Sampling $Q$**

Conditional on $B_t$, $Q$ was sampled from the inverse Wishart distribution with scale matrix $(B_t - B_{t-1})'(B_t - B_{t-1}) + Q_0$ and degrees of freedom $T + T_0$.

Estimation is based on 50,000 repetitions of steps 1 to 4 above, from which the last 10,000 draws were retained to build the posterior distributions of the parameters.